

All of the UK curricula define multiple categories of mathematical proficiency that require pupils to be able to use and apply mathematics, beyond simple recall of facts and standard procedures. While the intentions are very similar, the terminology varies between regions. *Progress Test in Maths' (PTS)* categories are based on the *Curriculum Aims* in the KS1, KS2 and KS3 National Curriculum for England (2013), and are also comparable with the GCSE Assessment Objectives: they adopt some language from both. The main change has been to divide 'Fluency' into two strands.

FF: Fluency in facts and procedures

Pupils can, for example:

- recall mathematical facts, terminology and definitions (such as the properties of shapes);
- recall number bonds and multiplication tables;
- perform straightforward calculations.

FC: Fluency in conceptual understanding

Pupils can, for example:

- demonstrate understanding of a mathematical concept in the context of a routine problem (for example, calculate with or compare decimal numbers, identify odd numbers, prime numbers and multiples) ;
- extract information from common representations, such as charts, graphs, tables and diagrams;
- identify and apply the appropriate mathematical procedure or operation in a straightforward word problem (for example, knowing when to add, multiply or divide).

MR: Mathematical reasoning

Pupils can, for example:

- make deductions, inferences and draw conclusions from mathematical information;
- construct chains of reasoning to achieve a given result;
- interpret and communicate information accurately.

PS: Problem solving

Pupils can, for example:

- translate problems in mathematical or non-mathematical contexts into a process or a series of mathematical processes;
- make and use connections between different parts of mathematics;
- interpret results in the context of the given problem;

- evaluate methods used and results obtained;
- evaluate solutions to identify how they may have been affected by assumptions made.

There is a limit to how thoroughly MR and PS can be assessed in a short, whole-curriculum test such as *PTM*, especially at younger ages where reading and English comprehension restrict the sorts of questions that can be asked. Teachers are urged to ensure that their curriculum includes a balanced diet of extended tasks, investigations, problem solving and collaborative activities.

This table shows how the questions in *PTM9* map onto these process categories.

Process category	Mental Maths	Applying and Understanding Maths
FF: Fluency in facts and procedures	1, 2, 5, 10, 12, 15	5, 6, 13, 15, 16a
FC: Fluency in conceptual understanding	3, 6, 9, 11	1, 2, 3, 7, 9, 10, 11, 12, 17, 18, 19, 20, 23, 28, 29
MR: Mathematical reasoning	4, 7, 8, 13, 14	4, 7, 8, 14a, 16b, 22, 24
PS: Problem solving		14b, 21, 25, 26, 27, 30

Mathematics process categories in Wales, Scotland and Northern Ireland

The process categories shown above are based on the National Curriculum and GCSE syllabuses for England. The curricula for Wales, Scotland and Northern Ireland have similar requirements, although there is wide variation in the way they are defined.

Wales	Closest <i>PTM</i> process categories			
Key Stage 2 Skills	FF	FC	MR	PS
1. Solve mathematical problems				•
2. Communicate mathematically		•	•	
3. Reason mathematically		•	•	
Foundation Phase Range	•			

Northern Ireland	Closest <i>PTM</i> process categories			
Key Stage 2 Processes in Mathematics	FF	FC	MR	PS
Making and monitoring decisions				•
Communicating mathematically		•	•	
Mathematical reasoning		•	•	•
Individual mathematical topics	•			

Scotland *	Closest PTM process categories			
Experiences and outcomes	FF	FC	MR	PS
develop a secure understanding of the concepts, principles and processes of mathematics and apply these in different contexts, including the world of work			•	•
engage with more abstract mathematical concepts and develop important new kinds of thinking			•	
understand the application of mathematics, its impact on our society past and present, and its potential for the future				
develop essential numeracy skills which will allow me to participate fully in society	•			
establish firm foundations for further specialist learning	•	•		
understand that successful independent living requires financial awareness, effective money management, using schedules and other related skills			•	•
interpret numerical information appropriately and use it to draw conclusions, assess risk, and make reasoned evaluations and informed decisions				•
apply skills and understanding creatively and logically to solve problems, within a variety of contexts			•	•
appreciate how the imaginative and effective use of technologies can enhance the development of skills and concepts				

* Education Scotland 'Curriculum for Excellence: Numeracy and Mathematics' 14 May 2009.
 Accessed: 31 July 2014. www.curriculumforexcellencescotland.gov.uk

Assessment for learning: following up the test activities

Each *PTM* assessment test is designed to align with the mathematics curriculum at a level appropriate for the pupils in the relevant age group. The activities may therefore be used to obtain diagnostic information about each pupil's strengths and weaknesses, and may also be used to provide a basis from which pupils' mathematical understanding may be further developed.

This section discusses some of the ways in which pupils may be helped to improve areas of weakness and to build on what they already know in order to deepen their understanding. These notes cover only a few of the possibilities. In talking to pupils and discussing the activities in which they did well, in addition to those they were unable to complete correctly, you may find approaches that are helpful to them, building on their own strengths and interests.

You will need to refer to the activities in the Pupil Booklet and the Teacher's script in the At a Glance Guide when reading these notes, as they form the basis of the ideas suggested. The activities are referred to here by both their numbers and their names.

Formative notes on the questions

The standardised total scores on *PTM* give you an indication of the overall performance of your pupils, and a basis for progress monitoring. This section is intended to help you identify the specific difficulties that pupils have with individual questions, and to suggest possible activities to help guide your future teaching.

Mental Maths test

These questions test pupils' basic number skills and recall of facts. If students score poorly, it may be that they simply lack these skills and are relying too heavily on written methods for even simple sums. They may lack the confidence to recall mathematical facts under time pressure.

Regular quick-fire quizzes may help pupils gain fluency and confidence, and there are many software products that allow students to practise skills in the context of games.

However, these should not displace problem-solving and investigative mathematics activities, which can also help pupils gain fluency by fostering a deeper understanding of mathematical concepts and their connections, reducing their dependence on 'memorising' fragments of information.

Paper and digital test

Question 1 and 2: Shaded fractions

In question 1, pupils are provided with eight rectangles shaded to show different fractions and they are asked to draw lines to match the pairs of equivalent fractions. Then, in question 2, pupils are shown a rectangle split into eight equal parts, a number of which are shaded; they are asked what fraction is shaded.

Pupils benefit from working on identifying fractions that add up to one whole and, perhaps, from counting up in fractions, which is also recommended in the non-statutory guidance. Additionally, see notes on question 7.

Question 3 and 4: Cold nights

In this question, pupils are shown a table of the temperatures at midnight during one week. Pupils are asked to find which day had the lowest midnight temperature (question 3a), which day had a midnight temperature one degree higher than Friday (question 3b) and to find a temperature five degrees higher than -2° (question 4).

This question involves the interpretation of positive and negative temperatures near to zero. Pupils should be encouraged to use aids such as diagrams of thermometers (sketching one of their own if necessary) to help avoid mistakes. In the classroom, pupils need to work with negative numbers in contexts such as temperature, scoring in games, heights above and below sea level, and gain experience of walking up and down number lines as well as moving up and down thermometers. Board games that use dice with positive and negative numbers can provide plenty of practice in an enjoyable setting. Pupils may also like to invent their own games.

Question 5 and 6: Window patterns

A diagram showing a pattern from an old stained glass window is shown. Pupils are asked to name three shapes within the window (question 5). They then must identify a shape in which all the angles are right angles (question 6a) and one in which all the angles are greater than a right angle (question 6b).

In the classroom, working with patterns such as this provides opportunities for producing beautiful designs with systematic colouring, and for pupils to identify equal sides and angles, do angle calculations, identify line and, perhaps, rotational symmetries for individual shapes and for the pattern as a whole, discuss transformations and so on – a rich and beautiful illustration of mathematics!

Question 7: Adding fractions

In this task, a sum of two fractions represented in a calculation and a circle diagram are given as an example to demonstrate fractions which add up to one whole. For the question, pupils are provided with two fraction sums that have numbers missing. For the first sum, pupils must fill in the missing numerator of one of the fractions and for the second sum, they are asked to fill in two missing denominators.

Different representations of fractions, such as fraction walls, number lines, circular and other diagrams, as seen in other parts of this test paper, can be used. A pair of circular paper plates of different colours cut along a radius can be slid together and one rotated on the other to show different pairs of fractions making up a whole. Additionally, see notes on questions 1 and 2.

Question 8: Athletics

In this question, some athletics events and the times taken to complete them are shown. Pupils are asked to match each event to the most appropriate length of time.

In the classroom, sport is a motivating context, and pupils can enjoy and benefit from exploring information such as Olympic records, as well as measuring and recording distances and times relating to their own performance in some small-scale athletics activities. Many pupils have a very poor knowledge of units and the sizes of everyday objects, so the more opportunity they have to use units in context the better.

Question 9-11: Tigers

This task tests pupils' ability to interpret data. Pupils are shown a bar chart detailing the number of tigers in different countries and are asked which country has the least tigers (question 9); roughly how many tigers there are in Malaysia (question 10) and to circle three countries that have less than 500 tigers (question 11).

Classroom experience of collecting and representing data of interest to pupils will increase understanding and success. Pupils can add interest to their work by using the data they have collected themselves to answer questions that interest them.

Question 12: Which number?

This question tests knowledge of place value; pupils are asked which number has 7 as its hundreds digit (part a); which number is ten times bigger than 62 (part b); which number is not a whole number (part c) and which is the smallest number (part d).

Pupils need much time and experience to build up their understanding of place value and its importance when multiplying and dividing by powers of ten. Practical

activities, such as pupils sitting in a row representing the different digits of a number and exploring what happens to each digit when the number is multiplied or divided by powers of ten, can be enjoyable and informative. Exploring such multiplications and divisions on a calculator can also lead to the identification and understanding of the patterns in the answers. Pupils could also complete written tasks such as:

$$4.2 \times 1 = 4.2$$

$$4.2 \times 10 = 42$$

$$4.2 \times 100 = 420$$

This would encourage them to see and explain the connection between one line and the next. Working with scaling also provides good practice with powers of ten.

Question 13 and 14: Number machine

A number machine that multiplies numbers by 2 and then subtracts 3 is shown. Pupils are asked what number comes out when 4 goes in (question 13); what number goes in when 9 comes out (question 14a) and what number comes out the same as it goes in (question 14b).

Pupils need to be encouraged to have a go, to try numbers and see what happens. It does not take them long to find the correct number! In the classroom, pupils can invent their own number machines and explore their effects. Pupils could try to invent, for example, number machines for which every number emerges the same as it goes in. Teachers might choose to encourage some pupils to explore an algebraic approach to such problems.

Question 15 and 16: Biggest and smallest

Four two-digit numbers are shown. Question 15a asks pupils to identify the biggest number, and 15b asks pupils to write a number bigger than the biggest number shown. Question 16a asks pupils which are the two smallest numbers and 16b asks pupils to write down a whole number between the two smallest numbers.

Pupils may like to explore other questions like this that have more than one possible correct answer. How many of the answers are even? Prime? Multiples of 5?

Question 17: 60p

Five amounts of money are shown and pupils are asked which amounts are equal to seventy pence.

Unfortunately, there is misuse of money notation in the real world, where, for example, what is meant as sixty pence is sometimes incorrectly expressed as 0.60p or £0.60p. Pupils need to explore and discuss such mistakes and correct them.

Question 18 - 20: Halfway

Each part of this question asks pupils to find a number exactly halfway between a pair of numbers. Pupils are asked to find the halfway point between a pair of whole numbers (question 18); to find the halfway point between a pair of decimal numbers (question 19) and to find the halfway point between a pair of fractions (question 20).

Using a number line can be helpful in all three cases, particularly in the case of the whole numbers. Pupils can be helped to understand (perhaps using multilink or similar equipment) that the halfway amount is what each person would get if the total were shared out equally between two people.

Question 21: Sorting shapes

Pupils are provided with a diagram for sorting shapes and are asked to sort seven polygons according to four different properties.

Any work that pupils can do in the classroom on classifying shapes in various ways (such as the use of sorting trees, Venn diagrams and tables such as in this question) will develop their expertise at managing a number of items of information simultaneously. The same activities can also be used with numbers rather than shapes, using categories such as multiples of three, numbers greater than ten, odd numbers and so on.

Question 22: Thirty-two

The multiplication sum $2 \times 16 = 32$ is shown. Pupils are asked to choose four different numbers to show two more multiplications with the answer 32.

In the classroom, pupils can start with a calculation such as $2 \times 18 = 36$ and use it to generate other calculations such as $4 \times 9 = 36$, $6 \times 6 = 36$ and so on, to gain understanding of the fact that, as one number gets smaller, the other must get bigger by the same factor to maintain the constant product; and, of course, to gain some valuable practice and facility with number, with and without a calculator.

Pupils should not restrict themselves to working with whole numbers. How many products are there with an answer of 36? Pupils could plot pairs of numbers with a product of 36 on a coordinate grid. There are also links with the areas of rectangles - which rectangle has the smallest/largest perimeter etc.? All of these activities will contribute to developing a deep understanding of the meaning of multiplication.

Question 23: Changing £10

Pupils must find how many twenty pence pieces there are in £10.

Teachers perhaps tend to assume that pupils have a good knowledge and understanding of money, but this is not always the case, at least not when questions about money appear in the classroom or in a test. Pupils might identify

how many 20p there are in £1, £2, £3 and so on, where there are number patterns to be explored and explained, and other similar conversions. There are also opportunities to use fractions and decimals. Teachers may wish to set such work in the context of a wider exploration of money matters and personal finance.

Question 24, 25 and 26: Number cards

Four number cards are shown and pupils are asked to make the smallest number possible using the four cards (question 24); to arrange the cards to make two numbers whose sum is 74 (question 25) and to arrange the cards to make two numbers whose difference is 26 (question 26).

In the classroom, pupils can discuss how such problems can be solved, asking questions such as how many possible sums can be made with four digits; how you can get a unit's digit of five using these digit cards; how the two cards in the tens place can add up to eight, and so on. In doing so, they will not only gain some useful practice using calculations, but will also do some mathematical reasoning and explanation.

Question 27: Invitations

Leah makes sixteen invitations for her party and each need four stickers; Leah has a sheet of eighty stickers. Question 27a asks pupils how many stickers are left once Leah has made all of the invitations and question 27b asks pupils how many more invitations she can make with the left-over stickers.

Solving mathematical problems is a requirement of everyday life, so pupils need to be given such problems to consider. They need to be encouraged to explore and evaluate different methods and to explain them to their peers. To be successful, they also need basic number skills, and such skills will develop if used in realistic contexts as part of solving problems. How can pupils quickly and easily calculate 16×5 ? Do they know it is the same as 5×16 ?

Question 28 - 30: Heartbeats

A table showing the usual number of heartbeats per minute for people of different ages is shown, together with a graph of the data for people at a medical centre on a particular day. In question 28, pupils are asked how many babies were at the medical centre; in question 29, pupils are asked to put a cross on the graph that shows John's heartbeat and in question 30 pupils are asked to say which two age groups Erin could be in, based on information given about her heartbeat.

As mentioned earlier, pupils need to work with data presented in different ways, and to generate and present their own data, so that they are able to work flexibly with data presented in real life situations and in tests.

Feedback to parents and carers

A report on the individual pupil is available to support feedback to parents or carers. This *Individual report for parents* strips away much of the technical detail that is included in the *Group report for teachers*. A series of statements, tailored for parents, is included to explain what the results mean and how learning may be affected. Recommendations focus on how the parent or carer can work with the school to support the pupil at home.

In addition to the *Individual report for parents*, you may wish to provide supporting information, either orally or in writing, explaining the process and outcomes. The following list provides you with guidelines to assist with this communication.

- Stress the school's commitment to identifying and addressing the needs of each individual pupil in order to understand and maximise their potential.
- Explain that testing with *PTS9* is part of the school's regular assessment regime and that all pupils in the year group(s) have been tested.
- As part of the test, pupils were tested on their mental maths ability as well as their ability to apply and understand mathematics in a written context.
- You may wish to summarise the specific outcomes and recommendations from the test for that individual pupil (which are also shown on the *Individual report for parents*).
- Parents or carers should be reassured that if they have any questions or concerns or would like any further advice on how best to support their child, then they should contact the school.

A sample letter (Figure 1) is provided to support your communications with parents/carers after testing with *PTS9*.

Figure 1: Sample parent/carer feedback letter

Dear Parent or Carer,

In school, we wish to assess all our pupils to see what their needs are and how we can best help them learn and achieve.

As part of this process, your child has completed the Progress Test in Maths 9, which assesses key aspects of maths, such as shape, number and mathematical concepts (like money, place value and time).

A copy of the Individual report for parents is included*. This shows your child's results and describes what these mean in terms of the ways in which he/she will learn best and how you can support him/her at home.

[If the report is not included a relevant short extract can be included instead.]

If you have any queries or concerns please contact us.

Yours faithfully,

[School/Establishment name]

* If possible, it is helpful to parents to discuss the report with them on a suitable occasion before sending it out.