Mathematics process categories Form B

PT MATHS 14

All of the UK curricula define multiple categories of mathematical proficiency that require students to be able to use and apply mathematics, beyond simple recall of facts and standard procedures. While the intentions are very similar, the terminology varies between regions. *Progress Test in Maths (PTM)* categories are based on the Aims in the KS1, KS2 and KS3 *National Curriculum for England*, and are also comparable with the GCSE Assessment Objectives, adopting some language from both. The main change has been to divide 'Fluency' into two strands.

FF: Fluency in facts and procedures

Students can, for example:

- recall mathematical facts, terminology and definitions (such as the properties of shapes);
- recall number bonds and multiplication tables;
- perform straightforward calculations.

FC: Fluency in conceptual understanding

Students can, for example:

- demonstrate understanding of a mathematical concept in the context of a routine problem (e.g. calculate with or compare decimal numbers, identify odd numbers, prime numbers, multiples);
- extract information from common representations, such as charts, graphs, tables and diagrams;
- identify and apply the appropriate mathematical procedure or operation in a straightforward word problem (for example, knowing when to add, multiply or divide).

MR: Mathematical reasoning

Students can, for example:

- make deductions, inferences and draw conclusions from mathematical information;
- construct chains of reasoning to achieve a given result;
- interpret and communicate information accurately.

PS: Problem solving

Students can, for example:

- translate problems in mathematical or non-mathematical contexts into a process or a series of mathematical processes;
- make and use connections between different parts of mathematics;
- interpret results in the context of the given problem;

- evaluate methods used and results obtained;
- evaluate solutions to identify how they may have been affected by assumptions made.

There is a limit to how thoroughly MR and PS can be assessed in a short, whole-curriculum test such as *PTM*. Teachers are urged to ensure that their curriculum includes a balanced diet of extended tasks, investigations, problem solving and collaborative activities.

This table shows how the questions in *PTM*14 map onto these process categories.

Process category	Mental Maths	Applying and Understanding Maths
FF: Fluency in facts and procedures	1, 2, 3, 9, 11, 12, 15	1
FC: Fluency in conceptual understanding	4, 5, 6, 7, 8, 10, 13, 14, 16, 17, 18, 19, 20	2, 3, 4, 5, 14, 16, 18, 19
MR: Mathematical reasoning		6, 7, 8, 9, 10, 11, 12, 13, 15, 17, 20, 22
PS: Problem solving		21, 23, 24

Mathematics process categories in Wales, Scotland and Northern Ireland

The process categories are based on the National Curriculum and GCSE syllabuses for England. The curricula for Wales, Scotland and Northern Ireland have similar requirements, although there is wide variation in the way they are defined.

Wales	Closest *PTM* process categories			
Key Stage 3 Skills	**FF**	**FC**	**MR**	**PS**
1. Solve Mathematical Problems				•
2. Communicate Mathematically		•	•	
3. Reason Mathematically		•	•	
Key Stage 3 Range	•			

Northern Ireland	Closest *PTM* process categories			
Key Stage 3 Using Mathematics	**FF**	**FC**	**MR**	**PS**
Communicate		•	•	
Manage Information			•	
Think Critically		•	•	
Solve Problems and Make Decisions				•
Individual mathematical topics	•			

Scotland	Closest *PTM* process categories			
Experiences and outcomes	**FF**	**FC**	**MR**	**PS**
develop a secure understanding of the concepts, principles and processes of mathematics and apply these in different contexts, including the world of work			•	•
engage with more abstract mathematical concepts and develop important new kinds of thinking			•	
understand the application of mathematics, its impact on our society past and present, and its potential for the future				
develop essential numeracy skills which will allow me to participate fully in society	•			
establish firm foundations for further specialist learning	•	•		
understand that successful independent living requires financial awareness, effective money management, using schedules and other related skills			•	•
interpret numerical information appropriately and use it to draw conclusions, assess risk, and make reasoned evaluations and informed decisions				•
apply skills and understanding creatively and logically to solve problems, within a variety of contexts			•	•
appreciate how the imaginative and effective use of technologies can enhance the development of skills and concepts				

Education Scotland: "Curriculum for Excellence: Numeracy and Mathematics" 14 May 2009.

Assessment for learning: following up the test activities

Each *PTM* assessment test is designed to align with the mathematics curriculum at a level appropriate for the students in the relevant age group. The activities may therefore be used to obtain diagnostic information about each student's strengths and weaknesses, and may also be used to provide a basis from which students' mathematical understanding may be further developed.

This section discusses some of the ways in which students may be helped to improve areas of weakness and to build on what they already know in order to deepen their understanding. These notes cover only a few of the possibilities. In talking to students and discussing the activities on which they did well, as well as those they were unable to complete correctly, you may find approaches that are helpful to them, building on their own strengths and interests.

You will need to refer to the activities in the *Student Booklet* and the Teacher's script in the *At a Glance Guide* when reading these notes, as they form the basis of the ideas suggested. The activities are referred to here by both their numbers and their names.

Formative notes on the questions

The standardised total scores on *PTM* give you an indication of the *overall* performance of your students, and a basis for progress monitoring. This section is intended to you help identify the specific difficulties that students have with individual questions, and to suggest possible activities to help guide your future teaching.

Mental Maths test

These questions test students' basic number skills and recall of facts. If students score poorly, it may be that they simply lack these skills, and are relying too heavily on written methods for even simple calculations. They may lack the confidence to recall mathematical facts under pressure.

Regular quick-fire quizzes may help students gain fluency and confidence, and there are many software packages that allow students to practice skills in the context of games.

However, these should not displace problem-solving and investigative mathematics activities, which can also help students gain fluency by fostering a deeper understanding of mathematical concepts and their connections, reducing their dependence on 'memorising' fragments of information.

Paper and digital test

Question 1: Fractions

In this question, students are firstly asked to add two fractions with different denominators and, as the total is greater than one, it needs to be expressed as a mixed number (part a). Then, students are asked to subtract a fraction and a mixed number, which again have different denominators (part b).

Many students find working with fractions difficult, so regular practice can be helpful. Using fractions and mixed numbers rather than decimals when studying other topics such as area and perimeter or averages can be useful.

Question 2: Calculation using negative exponents

This question presents students with four calculations including numbers with negative exponents. Students are asked to select which of the calculations is the largest.

Question 3: Standard form

This question will present students with an expression in the form of a fraction, in which both numerator and denominator are given in standard form. Students are asked to calculate the value of the expression in standard form.

Students should be encouraged to explore simplification methods, or to calculate the value of the numerator and denominator first, whilst ensuring they apply the laws of indices.

Question 4: Brackets

In this question, students are given an expression in the form of two brackets containing linear expressions. Students are asked to circle equivalent expressions from a list of a further six expressions. The list contains three equivalent expressions and three expressions which contain common errors that students make when expanding brackets.

Expanding products of binomials is a topic that students find quite difficult. Pairing up equivalent fractions and correcting incorrect 'homework' are approaches which can provide some variety when practising multiplying out brackets.

Question 5: Nets

In this task, three nets of cuboids are shown. Students are asked to pair up each of the three nets to the cuboid it makes from the four cuboids shown. Students need to recognise, describe and build simple 3D shapes, including making nets. In the

classroom, students need to practise cutting through the edges of 3D shapes to make nets before building a variety of 3D shapes from nets.

If students find this task difficult, they might first pair up the cube with the net that contains only square shapes. Then the net with small squares top and bottom could be matched with the 3D shape top right because it is the only 3D shape with two small square faces and four long narrow rectangular sides.

Question 6: Puppies

In this problem, students are given the weight of a puppy when it was born and asked to work out its weight at the end of the first and second months. Students are asked to increase the puppy's weight by a percentage (part a); they are then told that there is an increase of $\frac{3}{20}$ in the puppy's weight during the second month and must circle the appropriate method to calculate the increase in the puppy's weight (part b). Students are then asked for the answer for this fractional increase (part c).

Many students solve such problems by finding the percentage increase and adding it onto the original. As this is not a reversible process it is a good idea to go on to find the total new percentage in one step as this will help later when learning to find the original value given the value found after an increase or decrease.

Question 7: Perimeter Sequence

In this task, students are shown a table containing a sequence of four square shapes, increasing in size, with their shape number and perimeter. Students are asked to find the perimeter of the eleventh shape (part a) and are asked to write a formula for the perimeter of the nth term, based on a range of number and calculation cards (part b). Looking for the pattern to find how the shape 'grows' without finding all the terms in between can help students to find the nth term. After looking at the four shapes shown, students may see that the shape 'grows' by adding three sides of a square; this means that the perimeter increases in steps of 2cm because at each step 1cm is 'lost'.

When working on such tasks in the classroom, students need to be encouraged to describe how they see the shapes 'grow' – one student's vision may be very different to another student's vision. To get from one step of the sequence to the next, 2cm more is added to the perimeter. *N* represents the number in the sequence. As 2 is being added each time we are working with the two times table (2, 4, 6, 8 etc.) But this pattern starts at 4, two higher than the start of the two times table. Therefore, we have 2n +2.

Question 8 and 9: Swimming Race

Three graphs drawn on the same coordinate grid show the distance of each swimmer from the start line at different times during the race. In question 8,

students are asked which swimmer was slowest (part a), and how long she took (part b). In question 9 they are asked to find the time at which one swimmer overtook another.

Interpreting information when several graphs are drawn on the same axes can be quite difficult. A common error in responses to this question is to identify the winner as the person who took the longest time. Interpreting what graphs show and obtaining information from them are important skills which need practice in many different contexts. All too often students are asked to draw graphs, rather than discussing what information is shown in graphs that can be found in magazines and newspapers.

Question 10 and 11: Test Results

This task provides a scatter graph showing the scores for a group of students who all took two tests. In question 10a, students are asked to find the range of scores in Test A and in question 10b, they are first asked to find the student with the highest score on Test A, then find the score that they achieved on Test B. Question 11 gives students three statements relating to how the graph can be interpreted with some words missing, and students need to fill in the gaps using mathematical terms from a given list.

In the classroom, activities of this sort can help students realise how much information can be found when interpreting what a graph shows. In order to answer this type of question, both very careful reading and an understanding of the information shown in the graph are needed. It may be that students find this task difficult because they are not often asked to complete tasks of this sort.

Question 12 and 13: Sequence

In this question the first four terms of a number sequence are provided. In question 12a, students are asked to find the tenth term in the sequence and in part b, they are asked to use the number cards provided to write a formula for the *n*th term in the sequence. In question 13, students are presented with the start of a number sequence, as well as five 3-digit numbers. The numbers are the first four terms of an arithmetic sequence; the pattern is 'add 4'. Students are asked to choose which of the numbers would not appear in the sequence.

Question 14: Number Machine

Here we have a number machine made up of three functions: a subtraction, a squaring and another subtraction. Students are asked to work from left to right to find the result, *y*, of putting a value, *x*, through the machine if *x* is 5 (part a). Students are then asked to use inverse operations to find the value of the *x* if y is 0 (part b). Then, from a selection of equations, students are asked to choose which one this machine represents (part c).

Number machines are a useful way of leading into algebra. Reversing the process requires the use of opposite functions – in this case, subtraction with addition and squaring with square rooting. It is also necessary to use the functions in reverse order. Seeing this in a number machine can help towards learning to solve algebraic equations.

Question 15: Flower Garden

In this problem, students are presented with a diagram of a circular flower bed surrounded by a circular lawn. The diameter and the width of the lawn are given, together with the formula for the area of a circle and a value for pi. Students are asked to find the area of the flower bed correct to 1 decimal place (part a) and then they must find the area of the lawn (part b).

Before finding the area of the flower bed using the formula provided, students need to determine its radius by finding half of 8m and then subtracting 1m. They are also asked to write their answer correct to 1 decimal place. To find the area of the lawn, students need to calculate the area of the lawn when it includes the flower bed and then subtract the area of the flower bed.

Question 16: Poster

In this question, a picture of a yacht is shown together with its enlargement to make a poster. The height and width of the picture are shown, but only the width of the poster is given. Students are asked to find the height of the poster (part a); next, the height of the boat's mast on the poster is given and students are asked to calculate the height of the boat's mast in the picture (part b).

This question can be approached in various ways. Using the concepts of ratio or scale factor, by comparing the widths of the two diagrams, students may see that the poster is an enlargement of the picture by a scale factor of 1.5. This can then be used to find both the height of the poster, by multiplication, and the height of the mast, by division.

Question 17: Pace Length

In this problem, students are provided with a formula which shows the approximate relationship between the number of steps taken every minute and the pace length in metres. Students are asked to use this formula to fill in the missing numbers in the table, which shows different numbers of paces per minute and their corresponding pace lengths in metres.

To find the number of paces per minute students can rearrange the formula to get $n = P \times 140$. They can then do 0.45×140 to find n. To find the pace length they will need to rearrange to get $P = n/140$. If they have rearranged the first time, this may be more intuitive than if they look back at the original formula given.

Question 18: Translation

In this task students are presented with a co-ordinates grid with a flag. Students are asked to translate the flag four units to the right and one unit down.

Question 19: Scale factor

In this task students are presented with a co-ordinates grid with a flag. Students are asked to enlarge the shape by a scale factor of three.

Question 20: Toothpaste

This problem shows a cylinder of toothpaste and provides the formula for the volume of a cylinder, and the dimensions of this cylinder. The task is to find the volume of the toothpaste removed from the cylinder. The diameter of the toothpaste is given so this needs to be halved to find the radius needed in the formula. Both of the dimensions are less than a whole centimetre so the question uses small, decimal values. The answer is to be given correct to two decimal places.

Some students make errors that could have been easily avoided when multiplying and dividing by decimals. These errors can often be self-corrected if students are encouraged to ask themselves whether their answers make sense. In the classroom, this question could be opened out to find the volume of a tube of toothpaste and to find the length of time the tube could last.

Question 21: Roof

A diagram showing the cross section of a roof is given. The cross section is an isosceles triangle with its base length and vertical height given. The problem is to calculate the length of the slope of the roof.

In this problem students need to use Pythagoras' Theorem to calculate the hypotenuse. Students need to see that half of the diagram forms a right-angled triangle and that half the base length should be used in the calculation. This problem requires students to use Pythagoras' Theorem in its most straight forward form, but the numbers involved are decimals, so they need to take care when considering the positions of the decimal points as they square the sides and find the square root of the sum.

Question 22: School Trips

This task provides a tree diagram showing the different places that children will be going to on a school trip. At random, half of the group will be chosen to go to the countryside and half to the city; these probabilities are already entered on the diagram. Further information is given about the proportion of children visiting different places, and the question requires students to fill in the empty boxes on the probability tree (part a) and they are then asked to calculate the probability that a child will visit one particular place (part b).

At random, children are selected to visit three different places in the countryside and four different places in the city, so different fractions will need to be entered. Entering the probabilities in the boxes is comparatively easy: the fraction to be entered in the three top boxes is $\frac{1}{3}$ and $\frac{1}{4}$ in the bottom four boxes. The probability that Freya will go to the forest trail is more complex, as the fractions $\frac{1}{2}$ and $\frac{1}{3}$ need to be multiplied together to give $\frac{1}{6}$. Some students find calculating the probability of compound events difficult and tend to add the two fractions rather than multiplying them.

Question 23: Sorting Functions

In this task, diagrams of three graphs – two line graphs and one quadratic graph – are shown. Four equations are given: two are linear and two are quadratic, and students are asked to draw a line from each equation to its matching graph (part a) then work out which graph the table of values for *x* and *y* represents (part b).

This question has 2 linear and 1 quadratic graphs. Students first need to recognize and understand the difference between the equations of these graphs. There are 2 linear equations given, so once recognized, the students can use their understanding of either positive/negative gradients or the y-intercept to determine which linear equation matches which graph. The quadratic equation/ graph matching is a little trickier as the student will need to recognize the equation of a reflected $y = x^2$. They may also solve this by substituting coordinates to determine which equation matches the correct graph.

The second part of the question assesses a student's understanding of the relationship between *x* and *y* and squared numbers. Students should deduce that the *y* value is the negative value of *x* squared ($y = -x^2$).

Question 24: Hexagon

In this question, a regular pentagon with four diagonals is shown. Students are asked to calculate one of the interior angles (part a), to name two congruent triangles (part b) and to calculate the size of two of the angles formed by the diagonal (part c and d).

The interior angle of a regular hexagon can be found using a number of different approaches. One method requires student to know that the exterior angle of a regular hexagon is 360° ÷ 6 (60°) and that the sum of the angles on a straight line is 180°, so each interior angle is 120°. The diagonals already drawn show three congruent triangles. The sizes of the required angles can be calculated using the properties of the angles in an isosceles triangle.

Many geometry questions can be calculated using several methods. In the classroom, it is interesting to challenge different groups to use different methods, maybe giving them a rule which must be used, for example: 'the angles in a triangle add up to 180°' or 'the exterior angles of a polygon add up to 360°'.

Feedback to parents and carers

A report on the individual student is available to support feedback to parents or carers. This *Individual report for parents* strips away much of the technical detail that is included in the *Group report for teachers*. A series of statements, tailored for parents, is included to explain what the results mean and how learning may be affected. Recommendations focus on how the parent or carer can work with the school to support the student at home.

In addition to the *Individual report for parents*, you may wish to provide supporting information, either orally or in writing, explaining the process and outcomes. The following list provides you with guidelines to assist with this communication.

- Stress the school's commitment to identifying and addressing the needs of each individual student in order to understand and maximise their potential.
- Explain that testing with *PTM*14 is part of the school's regular assessment regime and that all students in the year group(s) have been tested.
- As part of the test, students were tested on their mental maths ability as well as their ability to apply and understand mathematics in a written context.
- You may wish to summarise the specific outcomes and recommendations from the test for that individual student (which are also shown on the *Individual report for parents*).
- Parents or carers should be reassured that if they have any questions or concerns or would like any further advice on how best to support their child, then they should contact the school.

A sample letter (Figure 1) is provided to support your communications with parents/carers after testing with *PTM*14.

Figure 1: Sample parent/carer feedback letter

Dear Parent or Carer,

In school, we wish to assess all our students to see what their needs are and how we can best help them learn and achieve.

As part of this process, your child has completed the Progress Test in Maths 14, which assesses key aspects of maths, such as shape, number and mathematical concepts (like money, place value and time).

A copy of the Individual report for parents is included*. This shows your child's results and describes what these mean in terms of the ways in which he/she will learn best and how you can support him/her at home.

[If the report is not included a relevant short extract can be included instead.]

If you have any queries or concerns please contact us.

Yours faithfully,

[School/Establishment name]

* If possible, it is helpful to parents to discuss the report with them on a suitable occasion before sending it out.