Mathematics process categories Form B

PT MATHS 13

All of the UK curricula define multiple categories of mathematical proficiency that require students to be able to use and apply mathematics, beyond simple recall of facts and standard procedures. While the intentions are very similar, the terminology varies between regions. *Progress Test in Maths (PTM)* categories are based on the Aims in the KS1, KS2 and KS3 *National Curriculum for England*, and are also comparable with the GCSE Assessment Objectives, adopting some language from both. The main change has been to divide 'Fluency' into two strands.

FF: Fluency in facts and procedures

Students can, for example:

- recall mathematical facts, terminology and definitions (such as the properties of shapes);
- recall number bonds and multiplication tables;
- perform straightforward calculations.

FC: Fluency in conceptual understanding

Students can, for example:

- demonstrate understanding of a mathematical concept in the context of a routine problem (e.g. calculate with or compare decimal numbers, identify odd numbers, prime numbers, multiples);
- extract information from common representations, such as charts, graphs, tables and diagrams;
- identify and apply the appropriate mathematical procedure or operation in a straightforward word problem (for example, knowing when to add, multiply or divide).

MR: Mathematical reasoning

Students can, for example:

- make deductions, inferences and draw conclusions from mathematical information;
- construct chains of reasoning to achieve a given result;
- interpret and communicate information accurately.

PS: Problem solving

Students can, for example:

- translate problems in mathematical or non-mathematical contexts into a process or a series of mathematical processes;
- make and use connections between different parts of mathematics;
- interpret results in the context of the given problem;

- evaluate methods used and results obtained;
- evaluate solutions to identify how they may have been affected by assumptions made.

There is a limit to how thoroughly MR and PS can be assessed in a short, whole-curriculum test such as *PTM*. Teachers are urged to ensure that their curriculum includes a balanced diet of extended tasks, investigations, problem solving and collaborative activities.

This table shows how the questions in *PTM*13 map onto these process categories.

Process category	Mental Maths	Applying and Understanding Maths
FF: Fluency in facts and procedures	2, 4, 10, 13, 14, 16	
FC: Fluency in conceptual understanding	1, 3, 5, 6, 7, 8, 9, 11, 12, 15, 17, 18, 19, 20	1, 2, 4, 10, 11
MR: Mathematical reasoning		3, 5, 6, 7, 8, 9, 12, 13, 14, 15, 16, 17, 18, 20, 21, 25, 26, 27, 28, 29
PS: Problem solving		19, 22, 23, 24, 30, 31

Mathematics process categories in Wales, Scotland and Northern Ireland

The process categories are based on the National Curriculum and GCSE syllabuses for England. The curricula for Wales, Scotland and Northern Ireland have similar requirements, although there is wide variation in the way they are defined.

Wales	Closest *PTM* process categories			
Key Stage 3 Skills	**FF**	**FC**	**MR**	**PS**
1. Solve Mathematical Problems				•
2. Communicate Mathematically		•	•	
3. Reason Mathematically		•	•	
Key Stage 3 Range	•			

Northern Ireland	Closest *PTM* process categories			
Key Stage 3 Using Mathematics	**FF**	**FC**	**MR**	**PS**
Communicate		•	•	
Manage Information			•	
Think Critically		•	•	
Solve Problems and Make Decisions				•
Individual mathematical topics	•			

Scotland	Closest *PTM* process categories			
Experiences and outcomes	**FF**	**FC**	**MR**	**PS**
develop a secure understanding of the concepts, principles and processes of mathematics and apply these in different contexts, including the world of work			•	•
engage with more abstract mathematical concepts and develop important new kinds of thinking			•	
understand the application of mathematics, its impact on our society past and present, and its potential for the future				
develop essential numeracy skills which will allow me to participate fully in society	•			
establish firm foundations for further specialist learning	•	•		
understand that successful independent living requires financial awareness, effective money management, using schedules and other related skills			•	•
interpret numerical information appropriately and use it to draw conclusions, assess risk, and make reasoned evaluations and informed decisions				•
apply skills and understanding creatively and logically to solve problems, within a variety of contexts			•	•
appreciate how the imaginative and effective use of technologies can enhance the development of skills and concepts				

Education Scotland: "Curriculum for Excellence: Numeracy and Mathematics" 14 May 2009.

Assessment for learning: following up the test activities

Each *PTM* assessment test is designed to align with the mathematics curriculum at a level appropriate for the students in the relevant age group. The activities may therefore be used to obtain diagnostic information about each student's strengths and weaknesses, and may also be used to provide a basis from which students' mathematical understanding may be further developed.

This section discusses some of the ways in which students may be helped to improve areas of weakness and to build on what they already know in order to deepen their understanding. These notes cover only a few of the possibilities. In talking to students and discussing the activities on which they did well, as well as those they were unable to complete correctly, you may find approaches that are helpful to them, building on their own strengths and interests.

You will need to refer to the activities in the *Student Booklet* and the Teacher's script in the *At a Glance Guide* when reading these notes, as they form the basis of the ideas suggested. The activities are referred to here by both their numbers and their names.

Formative notes on the questions

The standardised total scores on *PTM* give you an indication of the *overall* performance of your students, and a basis for progress monitoring. This section is intended to you help identify the specific difficulties that students have with individual questions, and to suggest possible activities to help guide your future teaching.

Mental Maths test

These questions test students' basic number skills and recall of facts. If students score poorly, it may be that they simply lack these skills, and are relying too heavily on written methods for even simple calculations. They may lack the confidence to recall mathematical facts under pressure.

Regular quick-fire quizzes may help students gain fluency and confidence, and there are many software packages that allow students to practice skills in the context of games.

However, these should not displace problem-solving and investigative mathematics activities, which can also help students gain fluency by fostering a deeper understanding of mathematical concepts and their connections, reducing their dependence on 'memorising' fragments of information.

Paper and digital test

Question 1: Fifths

This question asks students to write $\frac{1}{5}$ as a percentage (part a); write $\frac{1}{5}$ of a full turn as an angle (part b) and work out $\frac{1}{5}$ multiplied by 7 (part c).

Using fractions in different contexts provides an opportunity for students to recognise the connections between fractions, percentages, angles and mixed numbers. Classroom activities making up questions using other fractions within different contexts can provide further insights into these connections.

Question 2: Ordering

In this question students are asked to order numbers with up to three decimal places from smallest to largest. The numbers have one, two or three decimal places, so students need to understand that they should start by comparing the first decimal digit, then the second etc. Students may find it helpful to place zeros at the end of some of the decimal numbers as place holders so that all the numbers have the same number of digits after the decimal point. In the classroom, asking a variety of questions such as 'Which number has 6 in the hundredths position?' can help students to develop a deeper understanding of place value.

Question 3: Cubes

This question shows a diagram of a solid made using centimetre cubes: students are asked to find the volume of the solid. In order to answer this question, students need to understand that each cube has a volume of $1cm^3$ and, therefore, the volume is the same as the number of cubes.

Some students find it very difficult to interpret drawings of three-dimensional shapes, so classroom activities involving making solids that correspond to diagrams using actual cubes can be extremely helpful.

Question 4: Brackets

In this question students are asked to add brackets an equation to make it correct. Students need to be able to use brackets appropriately; the use of further classroom activities such as this illustrates the importance of knowing and using the correct order of operations and practising multiplication tables.

Question 5 - 6: Decimal Cards

Seven cards are provided on which are written brackets, operations and numbers. Students are asked to order the cards to produce calculations which result in the answer provided. Although these questions are similar to the last question, it is more difficult because some of the numbers are decimals.

Making up calculations using number cards can be an interesting activity in the classroom. Given a fixed set of cards, students can be asked to find the calculation which gives the biggest answer, the smallest answer, a prime number answer etc.

Question 7 - 9: Sale Prices

This question shows a graph comparing the original price with the sale price. Students are asked to fill in the label showing the sale price (question 7), complete the label showing the original price (question 8) and complete the label showing the sale price of an item with a price outside of the values on the graph (question 9). The first label requires the sale price to be found based on the original price, which is given in the question, but the second label is the other way round: the question gives the sale price, and the original price needs to be found. Therefore, care needs to be taken, and the student must make sure to read from the correct axis. The scales themselves must be read carefully, as a large square on each axis represents £20, so each small square represents £4. The last label has an original price which is outside the values on the graph, so a different value needs to be found and scaled up to get the required answer.

Reading off values from all sorts of graphs with different scales is an important skill and needs to be practised in the classroom.

Question 10: Day Care

In this problem, students need to divide a large number in a given ratio. Firstly, students are presented with five possible calculations for the problem and must circle two that are correct (part a). Then, students must work out the answer to this calculation (part b).

Calculating with large numbers, such as the one in this question, can worry some students, as often they practise using ratios with smaller values. Once an understanding of the concept of ratio has been established it is useful to solve problems with all sorts of numbers. The concept of ratio can be difficult to grasp. Using counters and putting them into groups to represent the required ratio, then adding up the totals, can be helpful for students.

Question 11: Equivalences

In this question, students are asked to put six numbers in order; two are percentages, two are fractions and two are decimals.

In order to do this students need to find equivalent decimals (or percentages) for the other values. Students need to understand and use the equivalences between fractions, decimals and percentages. Students can practice this by sorting a variety of given values and then move on to working out the values themselves. This is a skill which can be practised when doing other parts of the curriculum, if the

answer is required to be given in a particular form. In the classroom, problems such as 'Which is biggest, 60% of £100 or 0.7 of £100?' can stimulate interesting discussions.

Question 12: Advertising

This task presents a pie chart and a table of values showing percentages and angles in the pie chart. Students are asked to complete the table which has one of the percentages and two of the angles missing. They also need to calculate the total for both rows; they should be aware that this will be 100% and 360° given that this question is testing percentages and angles.

There are many ways of solving this task, but perhaps the easiest ways are:

- Find the percentage spent on Cars by adding the three given percentages and subtracting the total from 100
- Find the angle for Food by calculating 2.5% of 360°
- Find the angle for Other by making the total of the four angles in the pie chart 360°.

Students need to understand the relationship between angles in the pie chart and percentages. They also need to be able to draw and use accurately drawn pie charts as well as to calculate values using the table when the pie chart is not drawn accurately.

Question 13 - 14: Salaries

This question is about averages. Students are given a table showing the salaries for various groups of employees and a chief executive. Students are asked to find the modal salary (question 13) and to fill in the table to show what will happen to the mean and median salary if the chief executive's salary decreases (question 14).

Students have difficulty remembering which average is which, so an 'aide-memoire' can be useful. Students first learn how to find or calculate the required average but a deeper understanding is required in part b, in order to know what will happen if a value is changed. In the classroom, questions such as this, followed by discussion and explanation, can develop deeper student understanding.

Question 15: Pentagon

A pentagon with one line of symmetry is shown in this question and the size of two of its angles is given. From a choice of five different facts, students are asked to circle which one could be used to calculate the third missing angle (part a) and then they must calculate the missing angle (part b).

Whichever method is chosen to approach this question, students must first recognise the pairs of equal angles because of the line of symmetry. One method would require the student to know, or to calculate, the total for the interior angles of a pentagon. Otherwise, knowing the total for the exterior angles, and that the sum of the interior and exterior angle is 180°, could be an alternative method. Other useful methods split the pentagon, either down the line of reflection into two identical quadrilaterals, or by drawing a horizontal line to make a triangle and an isosceles trapezium.

Many geometry problems can be solved using several different methods. Interesting discussions in the classroom emerge when challenging different groups to find different solutions, maybe giving them a rule which must be used, for example: 'the sum of the angles in a triangle is 180°' or 'the sum of the exterior angles of a polygon is 360°'.

Question 16: Spinning

In this probability question we have two spinners with different numbers, and a chart showing some of the possible totals which can be scored when spinning both spinners. Students are asked to complete the addition chart to show all the possible totals (part a) and to work out the probability of Bill winning a prize (part b).

Part a requires an understanding of two-way tables as well as some care when doing simple addition. In order to find a correct answer to part b students need to carefully read the requirements for winning, and then calculate the probability using the appropriate algorithm. Situations such as this can provide challenging classroom problems. Having found the probability of winning, students may find it interesting to decide how much should be charged per 'go', and what size the prize should be in order to encourage lots of people to have a 'go' and still make money for charity on the game.

Question 17: Bronze

This question gives the ratio of copper to tin in bronze. Students are asked to work out how much tin is in a bronze bracelet weighing 260g (part a) and then to calculate how much a bronze bracelet weights altogether if it contains 99g of copper (part b).

Students need to be able to calculate using ratios. Many students find this a difficult topic. Students need to understand that there are 10 parts in the ratio (9+1). Then they can divide 260 by 10 to find out what one part of the ratio is worth. The final part is recognizing that tin represents only one part of the ratio (copper is worth 9) and so the answer is 26. Proportional reasoning is a key skill in the second part of the question and recognizing scale factors. Students can either use the relationship of 9 to 99 or the relationship of 9 to 1 to find the total amount of tin, before adding this to the total amount of copper to get the total weight.

Question 18: Equations

In this question we are given a pair of simultaneous equations. Students are asked to select the correct solution from a list of five pairs of values. Here are two different methods for finding the correct answer to this problem:

- The first method involves substituting the given values for x and y in both equations to see whether we get true statements. Use of this method can lead students to use trial and error whenever they have simultaneous equations to solve.
- The second method is an algebraic substitution method which can be used when no possible answers are listed. As both equations are in the form y = an expression in x, the two expressions must be equal so an equation in x can be formed and solved to get the correct value for x. This can then be substituted into either of the equations (in this case the second is simpler as it doesn't have a fraction) to find y. This method works for all pairs of linear equations given in this form and can lead to solving a linear and quadratic pair of equations.

An interesting classroom activity is to choose a pair of equations which don't have whole number answers and ask half the class to try to solve it using trial and error and the other half to solve it using algebraic substitution.

Question 19: Graphs

In this task, students are given four linear equations and a diagram showing three straight lines drawn on the coordinate *x/y* plane. The first part of the task asks students to write the correct equations on the three lines (part a); the second part asks students to use the remaining equation to work out where its line would cross the *y* axis (part b).

Students are taught equations of lines using the formula $y=mx+c$ where m represents the gradient and c the y intercept. The 5 in the first three equations tells us where the *y*-intercept is and students may need to rearrange the equations to find this. The final equation, $y = \frac{1}{5}x$ intercepts the y axis at (0, 0). Some students may also check using coordinates on each line to determine the equation of the line. The equation left is $y = x + 5$. Using $y=mx+c$ students can determine that the *y*-intercept is 5. Again, they could also determine this by finding y when $x = 0$.

Some students do not find it easy to identify the equations of lines that have been drawn. Particularly, when more than one graph has been drawn on the same axes. It is important to encourage students to discuss the properties of graphs after they have drawn them and, when given an equation, try to visualize what the graph will look like before drawing it.

Question 20: Photographs

This task concerns scale factors and the ratio of areas when the size of a photograph is reduced. Four photographs of the same size are to be reduced in size to fit all four onto a rectangle measuring 22 by 16cm. Students are asked to work out the width and height of each photo for the new layout (part a), give the scale factor of the reduction (part b) and find the ratio of the area of each original photo to its new area in the layout (part c).

In order to answer part a of this question, the original measurements need to be halved. In part b, students need to understand that the scale factor relates to the change in length and since the photograph gets smaller this will be a fractional scale factor, $\frac{1}{2}$ rather than 2. If students find the ratios of original areas and new areas, they should be encouraged to simplify these so that they realise the answer can be found more simply by looking at how many new photographs fit into the area of an original photograph. Alternatively, it may be that students understand that the ratio of the area is the square of the ratio of the length.

Question 21: Cycling Holiday

In this problem, students are asked to work out the distance a cyclist would travel each day given the time he plans to cycle and his average speed (part a). They then must work out how much time will be needed for another cyclist to travel at a particular number of miles per hour (part b).

An understanding that miles per hour means the miles travelled in one hour can help students work out whether to multiply or divide when working with speed, distance and time. A line/road drawn and divided into pieces with hours (1, 2, 3...) above and distance (12, 24, 36...) below can be a useful pictorial representation.

Remembering that Speed = $\frac{\text{Distance}}{\text{Time}}$ can be useful when solving problems such as this.

Question 22-24: Boxes

This is a question about number sequences. Students are shown the nets of three open boxes and are asked to complete the table beneath, which shows the number of squares per net, for Net 3 and Net 4 (question 22). Students are then asked to work out how many squares there would be in the tenth net (question 23) and write the formula for the number of squares in the nth net using the cards provided (question 24).

Students may find it difficult to visualise what the tenth net would look like, so working step by step from the given diagrams and the table is often useful. Discussion of how the diagrams change as they get bigger, and what the tenth net looks like without listing all the numbers in between can help students understand the process.

Question 25: Driving Schools

This problem compares the pass rate of two driving schools. Students need to calculate the number of people who passed last year (part a) and then demonstrate which school had the best results by circling the correct reason from a list of four (part b).

The first part requires the student to work out 60% of 50 and what 48 passes out of 75 learners is, as a percentage. An understanding that percent means per hundred and that fifty is half of one hundred is key to understanding the easiest way to working this out. In the second part, students need to realise that comparison must be made between either the percentage or the fraction of students that passed, not between the number of passes, as the total number of students at the two driving schools was different. Calculating 48 out of 75 gives us 64%, so Learn Plus had the best results last year.

Question 26-27: Cuboid

This question provides students with the formulae for the total length of a cube and the length of an internal diagonal, along with diagrams. Students are asked to calculate the total length of the edges of a cube with 4cm edges (question 26) and the diagonal length of a cube with edges 6cm long (question 27).

Although the formulae are given, students must first realise that a cube is just a special cuboid with all three dimensions the same, so in these questions a, b and c are all equal to 6. Students at this stage will probably not be familiar with these formulas but they should be able to substitute into them to calculate the required answers. It is not only necessary to substitute into the formulas correctly, but the operations need to be done in the correct order. The final step in question 27 is to give the answer correct to one decimal place.

Question 28-29: Mistakes

This task gives students an opportunity to 'be the teacher' by correcting an incorrect long multiplication (question 28) and a long division (question 29). The error in the long multiplication is one which many students make. When multiplying by 70, they forget to write a 0 in the units column, so actually multiply by 7. This is a conceptual error which students need to understand. The long division has a simple subtraction error. Many students struggle to be able to do long division and to find this error, they need to know how to do the process correctly.

Correcting incorrect calculations is a useful way of practising number work. Practising long multiplication and division when doing other parts of the curriculum can help to keep this skill fresh, rather than only revising it as a separate topic.

Question 30: Mowing a Lawn

This geometry task requires the use of Pythagoras' Theorem to find one of the sides of a triangle. Students need to read and understand the story about mowing a lawn with an electric mower and realise that they need to consider the triangle AEP even though the line PE is not drawn on the given diagram. Students are asked to choose the correct method to calculate the length of AE (part a). Students must use Pythagoras Theorem to calculate the length of AE to two decimal places (part b).

Students need practice at solving problems such as this. Working out the length of sides on diagrams, even when all that is needed is a subtraction (to find AE), is something some students find very difficult, especially if they need to interpret information given as a story.

Question 31: Bigger or Smaller

Here the students are given four expressions in n and asked, given that n is greater than zero, whether the expression is bigger or smaller than n or if it could be either, depending on the value of n. The answer for the first expression is given. The second expression divides n by 0.01; a misconception, that all divisions produce a smaller answer, is being challenged here. The third expression multiplies n by 0.01; students often assume that multiplication always gets an answer which is bigger, but again, this is not correct here. The fourth expression multiplies n by 100; this will probably be answered correctly by almost everyone.

In order to answer these questions correctly, students will find it helpful to substitute different values for n in each of the given expressions. Calculations using numbers other than positive numbers greater than 1 can surprise many students and should be experimented with; finding rules for them can be an interesting project.

Feedback to parents and carers

A report on the individual student is available to support feedback to parents or carers. This *Individual report for parents* strips away much of the technical detail that is included in the *Group report for teachers*. A series of statements, tailored for parents, is included to explain what the results mean and how learning may be affected. Recommendations focus on how the parent or carer can work with the school to support the student at home.

In addition to the *Individual report for parents*, you may wish to provide supporting information, either orally or in writing, explaining the process and outcomes. The following list provides you with guidelines to assist with this communication.

- Stress the school's commitment to identifying and addressing the needs of each individual student in order to understand and maximise their potential.
- Explain that testing with *PTM*13 is part of the school's regular assessment regime and that all students in the year group(s) have been tested.
- As part of the test, students were tested on their mental maths ability as well as their ability to apply and understand mathematics in a written context.
- You may wish to summarise the specific outcomes and recommendations from the test for that individual student (which are also shown on the *Individual report for parents*).
- Parents or carers should be reassured that if they have any questions or concerns or would like any further advice on how best to support their child, then they should contact the school.

A sample letter (Figure 1) is provided to support your communications with parents/carers after testing with *PTM13*.

Figure 1: Sample parent/carer feedback letter

Dear Parent or Carer,

In school, we wish to assess all our students to see what their needs are and how we can best help them learn and achieve.

As part of this process, your child has completed the Progress Test in Maths 13, which assesses key aspects of maths, such as shape, number and mathematical concepts (like money, place value and time).

A copy of the Individual report for parents is included*. This shows your child's results and describes what these mean in terms of the ways in which he/she will learn best and how you can support him/her at home.

[If the report is not included a relevant short extract can be included instead.]

If you have any queries or concerns please contact us.

Yours faithfully,

[School/Establishment name]

* If possible, it is helpful to parents to discuss the report with them on a suitable occasion before sending it out.