Mathematics process categories Form B

PT MATHS 12

All of the UK curricula define multiple categories of mathematical proficiency that require students to be able to use and apply mathematics, beyond simple recall of facts and standard procedures. While the intentions are very similar, the terminology varies between regions. *Progress Test in Maths (PTM)* categories are based on the Aims in the KS1, KS2 and KS3 *National Curriculum for England*, and are also comparable with the GCSE Assessment Objectives, adopting some language from both. The main change has been to divide 'Fluency' into two strands.

FF: Fluency in facts and procedures

Students can, for example:

- recall mathematical facts, terminology and definitions (such as the properties of shapes);
- recall number bonds and multiplication tables;
- perform straightforward calculations.

FC: Fluency in conceptual understanding

Students can, for example:

- demonstrate understanding of a mathematical concept in the context of a routine problem (e.g. calculate with or compare decimal numbers, identify odd numbers, prime numbers, multiples);
- extract information from common representations, such as charts, graphs, tables and diagrams;
- identify and apply the appropriate mathematical procedure or operation in a straightforward word problem (for example, knowing when to add, multiply or divide).

MR: Mathematical reasoning

Students can, for example:

- make deductions, inferences and draw conclusions from mathematical information;
- construct chains of reasoning to achieve a given result;
- interpret and communicate information accurately.

PS: Problem solving

Students can, for example:

- translate problems in mathematical or non-mathematical contexts into a process or a series of mathematical processes;
- make and use connections between different parts of mathematics;
- interpret results in the context of the given problem;

- evaluate methods used and results obtained;
- evaluate solutions to identify how they may have been affected by assumptions made.

There is a limit to how thoroughly MR and PS can be assessed in a short, whole-curriculum test such as *PTM*. Teachers are urged to ensure that their curriculum includes a balanced diet of extended tasks, investigations, problem solving and collaborative activities.

This table shows how the questions in *PTM*12 map onto these process categories.

Process category	Mental Maths	Applying and Understanding Maths
FF: Fluency in facts and procedures	1, 2, 8, 13, 14	1, 2, 3, 5, 6, 12, 13, 14
FC: Fluency in conceptual understanding	3, 4, 5, 6, 7, 9, 10, 11, 12, 15, 16, 17, 18, 19, 20	4, 7, 8, 9, 10, 11, 15, 17, 19, 21, 22
MR: Mathematical reasoning		16, 20, 23, 24, 26, 27, 28, 29
PS: Problem solving		18, 25, 30, 31, 32

Mathematics process categories in Wales, Scotland and Northern Ireland

The process categories are based on the National Curriculum and GCSE syllabuses for England. The curricula for Wales, Scotland and Northern Ireland have similar requirements, although there is wide variation in the way they are defined.

Wales	Closest *PTM* process categories			
Key Stage 3 Skills	**FF**	**FC**	**MR**	**PS**
1. Solve Mathematical Problems				•
2. Communicate Mathematically		•	•	
3. Reason Mathematically		•	•	
Key Stage 3 Range	•			

Northern Ireland	Closest *PTM* process categories			
Key Stage 3 Using Mathematics	**FF**	**FC**	**MR**	**PS**
Communicate		•	•	
Manage Information			•	
Think Critically		•	•	
Solve Problems and Make Decisions				•
Individual mathematical topics	•			

Scotland	Closest *PTM* process categories			
Experiences and outcomes	**FF**	**FC**	**MR**	**PS**
develop a secure understanding of the concepts, principles and processes of mathematics and apply these in different contexts, including the world of work			•	•
engage with more abstract mathematical concepts and develop important new kinds of thinking			•	
understand the application of mathematics, its impact on our society past and present, and its potential for the future				
develop essential numeracy skills which will allow me to participate fully in society	•			
establish firm foundations for further specialist learning	•	•		
understand that successful independent living requires financial awareness, effective money management, using schedules and other related skills			•	•
interpret numerical information appropriately and use it to draw conclusions, assess risk, and make reasoned evaluations and informed decisions				•
apply skills and understanding creatively and logically to solve problems within a variety of contexts			•	•
appreciate how the imaginative and effective use of technologies can enhance the development of skills and concepts				

Education Scotland: "Curriculum for Excellence: Numeracy and Mathematics" 14 May 2009.

Assessment for learning: following up the test activities

Each *PTM* assessment test is designed to align with the mathematics curriculum at a level appropriate for the students in the relevant age group. The activities may therefore be used to obtain diagnostic information about each student's strengths and weaknesses, and may also be used to provide a basis from which students' mathematical understanding may be further developed.

This section discusses some of the ways in which students may be helped to improve areas of weakness and to build on what they already know in order to deepen their understanding. These notes cover only a few of the possibilities. In talking to students and discussing the activities on which they did well, as well as those they were unable to complete correctly, you may find approaches that are helpful to them, building on their own strengths and interests.

You will need to refer to the activities in the *Student Booklet* and the Teacher's script in the *At a Glance Guide* when reading these notes, as they form the basis of the ideas suggested. The activities are referred to here by both their numbers and their names.

Formative notes on the questions

The standardised total scores on *PTM* give you an indication of the *overall* performance of your students, and a basis for progress monitoring. This section is intended to you help identify the specific difficulties that students have with individual questions, and to suggest possible activities to help guide your future teaching.

Mental Maths test

These questions test students' basic number skills and recall of facts. If students score poorly, it may be that they simply lack these skills, and are relying too heavily on written methods for even simple calculations. They may lack the confidence to recall mathematical facts under pressure.

Regular quick-fire quizzes may help students gain fluency and confidence, and there are many software packages that allow students to practice skills in the context of games.

However, these should not displace problem-solving and investigative mathematics activities, which can also help students gain fluency by fostering a deeper understanding of mathematical concepts and their connections, reducing their dependence on 'memorising' fragments of information.

Paper and digital test

Question 1 - 3: Properties of number

In this task, students are presented with eight numbers and are asked to circle all the numbers which are a multiple of 5 (question 1); all the prime numbers (question 2) and the numbers which are a factor of 24 (question 3).

Students need to read the question carefully, as the response to each question requires more than one answer, so the word 'numbers' rather than 'number' in the question should be noted. Students need to have an understanding of the concepts multiple, factor and prime number. Practice in the use of these concepts can be useful while working on other topics. The use of a multiplication square can help students to identify which numbers satisfy each of the requirements.

Question 4: Clocks

In this question students are asked to match digital and analogue clocks which show the same time. Care needs to be taken as two of the analogue clocks show the same time; the background colour showing day or night needs to be noted in order to make the distinction between a.m. and p.m.

Some students find reading a digital 24-hour clock difficult, but it is necessary to be able to read, write and convert time between analogue and digital 12 and 24-hour clocks when reading timetables that state times using the 24-hour notation. Having an analogue and a digital clock available in the classroom can be useful as many clocks show digital time nowadays.

Question 5: Matches

Students are shown five four-digit numbers, in the context of people attending a football match. Students are asked to order the numbers from lowest to highest (part a) and to calculate the decrease in attendance between two of the seasons from the table (part b).

Working in a context makes the question more relevant to the real world and motivates careful reading of the question. Using real data, such as that relating to a local football team, makes practice of this sort of number work more interesting in the classroom.

Question 6: Distances

In this task students are required to add and subtract simple fractions in the context of distances between villages. Students are asked to add two distances presented as fractions (part a) and to subtract two distances presented as fractions (part b).

Many students find manipulating fractions difficult, so frequent practice can be helpful. Using fractions rather than decimals for lengths, weights etc. in other parts of the curriculum can provide helpful practice. In this question only halves and quarters are involved. The use of visual aids such as fraction walls, number lines showing fractions, and shapes divided into equal parts, can be beneficial.

Question 7 and 8: Thermometer

In question 7, students are asked to read a negative value on a thermometer and in question 8 are asked to mark a negative value on a second thermometer scale.

To do this correctly students need to count down to the left through zero rather than up to the right. Using negative numbers in a familiar context such as temperature can make them more easily understood. Practice with scales which are vertical as well as horizontal is worthwhile. Everyday situations such as heights above and below sea level, and quiz games using positive and negative scores can add interest to work on this topic.

Question 9, 10 and 11: Coins

This task provides a table of information showing the diameter, thickness and weight of euro coins. Students must identify which coin has the largest diameter (question 9); find out how much the lightest coin weighs (question 10) and work out which two coins weigh 6.98 grams when put together (question 11).

Extracting the correct information from a table is a very useful skill and can be practised in the classroom using real tables such as bus timetables or information on the internet. This question also requires an ability to be accurate about which value is being asked for. The answer to 'which coin has the largest diameter?' needs students to identify the value of the thickest coin, whereas, in question 10 the answer is the weight of the lightest coin. The numbers provided in the table are given to two decimal places, so, in question 11, the addition of decimals is also being assessed.

Question 12 and 13: Triangles

These questions provide students with six triangles on a grid: in question 12, students must match the triangles with the name that best describes them; in question 13a, students need to find the reflection of triangle E and in question 13b, they need to find the translation of triangle E.

Some students find it difficult to recognise shapes in different orientations as being of the same type, so encouraging students to practise naming and describing triangle types whenever triangles in complex diagrams occur can be worthwhile. Some students find recognising translations and reflections of shapes quite difficult and benefit from using tracing paper in the early stages. Searching

for particular shapes in complex patterns, or making tangrams, can be fun ways of practising naming and describing shapes as students explore identifying translations and reflections of particular shapes.

Question 14: Film Year

This question features two images of the end of a film, which displays the year it was produced in Roman numerals. In part a, students must circle which of the films was made most recently, and in part b, students are asked to write the decimal number equivalent of the more recent film.

This task presents two TV screens showing dates in Roman numerals; another real-life place to find Roman numerals is on clock faces. Finding the answers to simple sums written in Roman numerals can be fun as a classroom activity and can emphasise why our present-day number system is preferable. Further classroom activities include writing the dates of birth of the students in different classes, or famous people, and the dates of historic events in Roman numerals.

Question 15: Jam

This questions present student with two measures of jam, one of which is 'regular' and one 'low-sugar'. Students are asked to find 45% of 340 grams (part a) and to write 85 grams as a percentage of 340 grams (part b).

Percentage is a topic which some students struggle with, but it is an important topic to master. Students need to understand that percentages, decimals and fractions are different ways of expressing proportions. Understanding that percentage means per hundred is the key to this topic. Practising writing percentages as fractions with denominator 100, and as decimals should be done in real life situations such as in this question: students need to realise that 55% can be written as 55/100 or 0.55.

Question 16: Rectangle

This question requires students to be able to plot points and draw shapes on the coordinate plane. They are given one pre-plotted point and are then asked to plot a further two co-ordinates given (part a); then, to plot a fourth point to create a rectangle (part b) and finally, to write the co-ordinates of this last plotted point (part c).

A frequent error which students make when plotting coordinates is to reverse their order. A useful aid to memory is that x comes before y in the alphabet, as it does in the coordinates. Some students find it difficult to recognise shapes when they are in different orientations, and this shape may not look like part of a rectangle to some students. Practice in drawing shapes with different angles and identifying shapes drawn in different orientations is extremely beneficial. Plotting points and connecting them to make interesting shapes can be a fun classroom activity.

Question 17: Number Line

This task asks students to show where four fractions fit on a decimal number line.

Finding equivalent fractions, with powers of ten as the denominator, is useful for many conversions from fraction to decimal. For example: $\frac{1}{2} = \frac{5}{10} = 0.5$, $\frac{2}{5} = \frac{4}{10} = 0.4$ and $\frac{1}{8} = \frac{125}{1000} = 0.125$.

Understanding that $\frac{1}{4}$ is half of $\frac{1}{2}$ and $\frac{1}{8}$ is half of $\frac{1}{4}$ etc. is important. Dividing a piece of paper into two, i.e., halving it, then halving each piece to get quarters and so on, can be a valuable visual aid.

Having understood halves, quarters and so on, it is important to also look at thirds, fifths etc. as some students get confused and think that all fractions can be found by repeated halving. Thinking of fractions as division calculations can be helpful for more difficult decimal equivalents. Converting $\frac{1}{9}$, $\frac{2}{9}$, $\frac{3}{9}$ and so on into equivalents by dividing the numerator by the denominator can be fun, as it creates an interesting series of decimal values, 0.111…, 0.222…, 0.333… and so on.

Question 18: Perimeter

In this question students are shown an image of a shape made up of different rods of two unknown lengths. Students are asked to write a formula for the perimeter of the shape in terms of x and y (part a); they must then use this formula to calculate the perimeter when $x = 3$ and $y = 4$ (part b) and then find what the lengths of the two rods would be if the perimeter were 30 metres.

If students find using x and y confusing, using different letters may be helpful. Using different numbers and discussing how to find the perimeter, then putting in the letters and doing the same process can lead to deeper understanding of why symbols are used in algebra.

Question 19: Heights

Students are given an ordered list showing the height of fifteen children and are asked to find which is the median height (part a) and which height is the mode (part b).

Students often mix up median, mode and mean so they may need some help remembering which is which. ‘Mode is the most frequent’ can be a useful reminder. In this question, the numbers are already ordered but this will not always be the case, so students should practise ordering sets of numbers relating to real life situations that interest them, such as their own heights, before finding the median and mode. In the classroom students should be encouraged to collect or find their own data to analyse; for example, they may choose to find their favourite flavour crisps or hot drink.

Question 20: Finding Angles

In this question, students work out the missing angles in a diagram using the rules for angles on a straight line (part a) and angles in an isosceles triangle (part b).

This diagram shows a triangle with an extended straight line. Some students find it difficult to work with such diagrams and need to practise using more complex shapes. A frequent error is a misunderstanding of the rule 'angles on a straight line add up to 180°'. In this diagram some students may incorrectly add together both of the angles at R and the interior angle at P to total 180° since all of these angles are on the same straight line. Students need to understand that the rule refers to the angles at a single point on a straight line.

Students also need to be familiar with working with diagrams that are 'not drawn accurately' where they are told which sides and angles are the same size rather than those that look the same on the diagram. Some students may also find an isosceles triangle in this orientation a problem, because they normally see isosceles triangles standing on their 'base'.

Question 21 and 22: Sports Clubs

This question is about ratios in the context of an athletics club and a swimming club. Students are given the ratio of girls to boys and the number of girls; from this information, students are asked to find the number of boys based on the number of girls (question 21a). They are also asked to work out what the ratio would be if the number of boys and girls both doubled (question 21b). Then, students are given a total number of people at a club and asked to find how many are boys and how many are girls, based on the ratio provided (question 22).

Using different coloured counters for boys and girls, or circles drawn on a piece of paper with the ratio written in each circle, can provide a useful visual aid when working with ratio tasks like this. Seeing a 5:3 ratio as groups of five red counters with three blue counters, for example, until they are all used up, can be very helpful. For question 22, we would begin with making groups of one blue counter (girls) and two red counters (boys) until there is a total of eighteen red and blue counters altogether.

Question 23 and 24: Sandwiches

This question is about identifying the correct pie chart from a table of values. Within the table, only three of the four pie chart angles are given. In question 23, students are asked to work out the remaining angle and fill in the total. In question 24, students are required to work out which of four pie charts is the correct one for this data.

This question requires students to know that the sum of the four angles in the pie chart must be 360°. However, students should also learn how to find the required angles for the given data if they need to draw a pie chart. An understanding that

the correct pie chart has the largest angle for Cheese (120°), and smallest angle for Chicken (69°) is needed to select the correct answer for the second part of this question.

Question 25: Sticks

This task looks at an increasing pattern made from sticks. Students are told that the number of sticks in the first pattern is 7 and they are required to count the number of sticks in patterns 2 and 3 before writing their results in a table. After working out the pattern in the number of sticks and writing in the numbers of sticks for patterns 2 and 3, students are asked to predict the number of sticks in patterns 4 and 5 (part a) and pattern 10 (part b). Students must then work out which pattern number would have 92 sticks (part c).

Having real sticks to make patterns can help those having problems with this type of question. Noting how many extra sticks are needed to make the next pattern is the key to solving this problem. Looking at the number of 'add on' sticks and taking them off the first pattern to see what the starting point was can also be a useful strategy. However, relying on using real sticks is not recommended when solving the final part of this question.

Question 26 and 27: Special Offer

This question is based around a special offer at a bookshop. Question 26 asks students to choose the calculation which would allow Ryan to calculate the saving he made, as a fraction of what he would have paid without the offer. Question 27a asks students to find what fraction of the total price has been saved in its simplest form and question 27b asks students what percentage of the normal total price he pays.

Many students find it difficult to know what to do when faced with this sort of question. It can help to read it through with them and point out the most important parts: "cheapest one … free" and "fraction of the normal total price". Finding and then using these two values to make a fraction is a logical approach. The idea of using pence rather than pounds, and so removing the need to work with decimal numbers, can be a useful strategy. The final step could be to simplify the fraction or change the decimal answer (on the calculator) into a fraction.

Question 28: Logo

In this question students are given two diagrams and are asked to find the missing angles. In order to work out the missing angle of the first diagram, students must recognise that a, b and c are three angles around a point, and know that the sum of angles around a point is 360° (part a). For the next diagram, students must recognise that the internal angles of a quadrilateral total to 360° and use known angles to find the missing angle (part b).

Students need to practise drawing shapes and diagrams using given measurements and angles accurately, before calculating and measuring lengths and angles which are not provided.

Question 29: Wheels

This question asks students to calculate the circumference of a wheel using the given formula and the diameter. They then need to round their answer to the nearest whole centimetre.

Becoming familiar with the use of ϖ in the formulas for finding the circumference and area of a circle is important. Working with circle formulas in situations such as this, which are relevant to students' everyday life, provides motivation for them. Students need to practise rounding to required degrees of accuracy as a final step in many questions, since they need to realise that it does not make sense to measure more accurately in many real-life situations.

Question 30: Ship Speed

In this question students are given a graph relating speed in knots and speed in kilometres per hour. Students are provided with a speed in knots, and must use the graph to convert this into speed in kilometres per hour (part a). Students are then provided with a speed in knots, which is not on the graph, to convert into speed in kilometres per hour (part b).

Some students find it difficult to read off values of the variables which are between the values labelled on the axes. They may need practice in determining accurately what each small square on each axis represents. In this question, each small square on each axis represents one unit. Another common error in questions such as this is to confuse the two axes of the graph.

Some students may also need help when deciding how to find conversions of values which are beyond the scope of the given graph (part b). They need to learn that a good strategy is to read off a smaller value on the graph, which can then be multiplied (scaled up) to get the required answer, rather than trying to draw an extension of the line on the graph. In this question, 45 knots is beyond a value shown on the straight line graph. Students may choose to read off the value for 5 or 15 knots and multiply by 9 or 3, to find an estimate of the correct value in kilometres per hour.

Question 31 and 32: Spinners

This question is about the possible outcomes when spinning two spinners. Students are asked to work out the total scores and order them from least likely to most likely in question 31, and in question 32 they must find the probability of getting a particular score.

Since the questions ask about total scores, students could be encouraged to add total values to the table. Playing games with dice or cards is a good way of practising probabilities. In the classroom, one interesting activity is to use ten large cards numbered 1 to 10, turn the cards over one at a time and put them where the class can see them; then ask the class to work out the probability that the next card will be higher or lower than the last card turned. This is fun and quite challenging since the denominator changes on each 'go'.

Feedback to parents and carers

A report on the individual student is available to support feedback to parents or carers. This *Individual report for parents* strips away much of the technical detail that is included in the *Group report for teachers*. A series of statements, tailored for parents, is included to explain what the results mean and how learning may be affected. Recommendations focus on how the parent or carer can work with the school to support the student at home.

In addition to the *Individual report for parents*, you may wish to provide supporting information, either orally or in writing, explaining the process and outcomes. The following list provides you with guidelines to assist with this communication.

- Stress the school's commitment to identifying and addressing the needs of each individual student in order to understand and maximise their potential.
- Explain that testing with *PTM*12 is part of the school's regular assessment regime and that all students in the year group(s) have been tested.
- As part of the test, students were tested on their mental maths ability as well as their ability to apply and understand mathematics in a written context.
- You may wish to summarise the specific outcomes and recommendations from the test for that individual student (which are also shown on the *Individual report for parents*).
- Parents or carers should be reassured that if they have any questions or concerns or would like any further advice on how best to support their child, then they should contact the school.

A sample letter (Figure 1) is provided to support your communications with parents/carers after testing with *PTM*12.

Figure 1: Sample parent/carer feedback letter

Dear Parent or Carer,

In school, we wish to assess all our students to see what their needs are and how we can best help them learn and achieve.

As part of this process, your child has completed the Progress Test in Maths 12, which assesses key aspects of maths, such as shape, number and mathematical concepts (like money, place value and time).

A copy of the Individual report for parents is included*. This shows your child's results and describes what these mean in terms of the ways in which he/she will learn best and how you can support him/her at home.

[If the report is not included a relevant short extract can be included instead.]

If you have any queries or concerns please contact us.

Yours faithfully,

[School/Establishment name]

* If possible, it is helpful to parents to discuss the report with them on a suitable occasion before sending it out.