Mathematics process categories Form A

All of the UK curricula define multiple categories of mathematical proficiency that require students to be able to use and apply mathematics, beyond simple recall of facts and standard procedures. While the intentions are very similar, the terminology varies between regions. *Progress Test in Maths (PTM)* categories are based on the Aims in the KS1, KS2 and KS3 *National Curriculum for England*, and are also comparable with the GCSE Assessment Objectives, adopting some language from both. The main change has been to divide 'Fluency' into two strands.

FF: Fluency in facts and procedures

Students can, for example:

- recall mathematical facts, terminology and definitions (such as the properties of shapes);
- recall number bonds and multiplication tables;
- perform straightforward calculations.

FC: Fluency in conceptual understanding

Students can, for example:

- demonstrate understanding of a mathematical concept in the context of a routine problem (e.g. calculate with or compare decimal numbers, identify odd numbers, prime numbers, multiples);
- extract information from common representations, such as charts, graphs, tables and diagrams;
- identify and apply the appropriate mathematical procedure or operation in a straightforward word problem (for example, knowing when to add, multiply or divide).

MR: Mathematical reasoning

Students can, for example:

- make deductions, inferences and draw conclusions from mathematical information;
- construct chains of reasoning to achieve a given result;
- interpret and communicate information accurately.

PS: Problem solving

Students can, for example:

- translate problems in mathematical or non-mathematical contexts into a process or a series of mathematical processes;
- make and use connections between different parts of mathematics;
- interpret results in the context of the given problem;

- evaluate methods used and results obtained;
- evaluate solutions to identify how they may have been affected by assumptions made.

There is a limit to how thoroughly MR and PS can be assessed in a short, whole-curriculum test such as *PTM*. Teachers are urged to ensure that their curriculum includes a balanced diet of extended tasks, investigations, problem solving and collaborative activities.

These tables show how the questions in *PTM*13 map onto these process categories.

Paper test		
Process category	**Mental Maths**	**Applying and Understanding Maths**
FF: Fluency in facts and procedures	2, 4, 10, 13, 14, 16	
FC: Fluency in conceptual understanding	1, 3, 5, 6, 7, 8, 9, 11, 12, 15, 17, 18, 19, 20	1, 2, 4, 7, 8
MR: Mathematical reasoning		3, 5, 6, 9, 10, 11, 12, 13, 14, 16, 17, 19, 20, 21
PS: Problem solving		15, 18, 22, 23

Digital test		
Process category	**Mental Maths**	**Applying and Understanding Maths**
FF: Fluency in facts and procedures	2, 4, 10, 13, 14, 16	
FC: Fluency in conceptual understanding	1, 3, 5, 6, 7, 8, 9, 11, 12, 15, 17, 18, 19, 20	1a, 1b, 1c, 2, 4, 10a, 10b, 11
MR: Mathematical reasoning		3, 5, 6, 7, 8, 9, 12, 13, 14, 15a, 15b, 16a, 16b, 17a, 17b, 18, 20a, 20b, 20c, 21a, 21b, 25a, 25b, 26, 27, 28, 29
PS: Problem solving		19a, 19b, 22, 23, 24, 30a, 30b, 31

Mathematics process categories in Wales, Scotland and Northern Ireland

The process categories are based on the National Curriculum and GCSE syllabuses for England. The curricula for Wales, Scotland and Northern Ireland have similar requirements, although there is wide variation in the way they are defined.

Wales	Closest *PTM* process categories			
Key Stage 3 Skills	**FF**	**FC**	**MR**	**PS**
1. Solve Mathematical Problems				•
2. Communicate Mathematically		•	•	
3. Reason Mathematically		•	•	
Key Stage 3 Range	•			

Northern Ireland	Closest *PTM* process categories			
Key Stage 3 Using Mathematics	**FF**	**FC**	**MR**	**PS**
Communicate		•	•	
Manage Information			•	
Think Critically		•	•	
Solve Problems and Make Decisions				•
Individual mathematical topics	•			

Scotland	Closest *PTM* process categories			
Experiences and outcomes	**FF**	**FC**	**MR**	**PS**
develop a secure understanding of the concepts, principles and processes of mathematics and apply these in different contexts, including the world of work			•	•
engage with more abstract mathematical concepts and develop important new kinds of thinking			•	
understand the application of mathematics, its impact on our society past and present, and its potential for the future				
develop essential numeracy skills which will allow me to participate fully in society	•			
establish firm foundations for further specialist learning	•	•		
understand that successful independent living requires financial awareness, effective money management, using schedules and other related skills			•	•
interpret numerical information appropriately and use it to draw conclusions, assess risk, and make reasoned evaluations and informed decisions				•
apply skills and understanding creatively and logically to solve problems, within a variety of contexts			•	•
appreciate how the imaginative and effective use of technologies can enhance the development of skills and concepts				

Education Scotland: "Curriculum for Excellence: Numeracy and Mathematics" 14 May 2009.

Assessment for learning: following up the test activities

Each *PTM* assessment test is designed to align with the mathematics curriculum at a level appropriate for the students in the relevant age group. The activities may therefore be used to obtain diagnostic information about each student's strengths and weaknesses, and may also be used to provide a basis from which students' mathematical understanding may be further developed.

This section discusses some of the ways in which students may be helped to improve areas of weakness and to build on what they already know in order to deepen their understanding. These notes cover only a few of the possibilities. In talking to students and discussing the activities on which they did well, as well as those they were unable to complete correctly, you may find approaches that are helpful to them, building on their own strengths and interests.

You will need to refer to the activities in the *Student Booklet* and the Teacher's script in the *At a Glance Guide* when reading these notes, as they form the basis of the ideas suggested. The activities are referred to here by both their numbers and their names.

Formative notes on the questions

The standardised total scores on *PTM* give you an indication of the *overall* performance of your students, and a basis for progress monitoring. This section is intended to you help identify the specific difficulties that students have with individual questions, and to suggest possible activities to help guide your future teaching.

Mental Maths test

These questions test students' basic number skills and recall of facts. If students score poorly, it may be that they simply lack these skills, and are relying too heavily on written methods for even simple calculations. They may lack the confidence to recall mathematical facts under pressure.

Regular quick-fire quizzes may help students gain fluency and confidence, and there are many software packages that allow students to practice skills in the context of games.

However, these should not displace problem-solving and investigative mathematics activities, which can also help students gain fluency by fostering a deeper understanding of mathematical concepts and their connections, reducing their dependence on 'memorising' fragments of information.

Paper Test

Question 1: Quarters

This question asks students to write $\frac{1}{4}$ as a percentage (part a), what angle is a $\frac{1}{4}$ turn (part b) and work out $\frac{1}{4}$ of 6 (part c).

Using fractions in different contexts provides an opportunity for students to recognise the connections between fractions, percentages, angles and mixed numbers.

Classroom activities making up questions using other fractions, such as $\frac{1}{3}$, within different contexts can provide further insights into these connections.

Question 2: Ordering

In this question students are asked to order numbers with up to three decimal places from smallest to largest. The numbers have one, two or three decimal places, so students need to understand that they should start by comparing the first decimal digit, then the second etc.

Students may find it helpful to add extra zeros at the end of some of the decimal numbers so that all the numbers have the same number of digits after the decimal point.

In the classroom, asking a variety of questions such as 'Which number has 6 in the hundredths position?' can help students to develop a deeper understanding of place value.

Question 3: Cubes

This question shows a diagram of a solid made using centimetre cubes: students are asked to find the volume of the solid. In order to answer this question, students need to understand that each cube has a volume of $1cm^3$ and, therefore, the volume is the same as the number of cubes.

Some students find it very difficult to interpret drawings of three dimensional shapes, so classroom activities involving making solids that correspond to diagrams using actual cubes can be extremely helpful.

Question 4: Brackets

In this question students are asked to add brackets to both sides of an equation, so that they work out to the same value.

Students need to be able to use brackets appropriately.

The use of further classroom activities such as this illustrates the importance of knowing and using the correct order of operations and practising multiplication tables.

Question 5: Decimal Cards

Seven cards are provided on which are written brackets, operations and numbers. Students are asked to order the cards to produce calculations which result in given answers. Although this question is similar to the last question, it is more difficult because some of the numbers are decimals.

Making up calculations using number cards can be an interesting activity in the classroom. Given a fixed set of cards, students can be asked to find the calculation which gives the biggest answer, the smallest answer, a prime number answer etc.

Question 6: Sale Prices

This question shows a graph comparing the original price with the sale price. Students are asked to complete the labels on three items, so that each label shows the original price and the sale price.

The first label requires the sale price to be found given the original price but the second label is the opposite way round, it gives the sale price and the original price needs to be found, so care needs to be taken reading from the correct axis. The scales themselves need careful reading as a large square on each axis represents £20, so each small square represents £4.

The last label has an original price which is outside the values on the graph so a different value needs to be found and scaled up to get the required answer.

Reading off values from all sorts of graphs with different scales is an important skill and needs to be practised in the classroom.

Question 7: Day Care

In this problem students need to divide a large number in a given ratio. They are asked to show their working.

Calculating with large numbers, such as the one in this question, can worry some students, as often they practice using ratios with smaller values. Once an understanding of the concept of ratio has been established it is useful to solve problems with all sorts of numbers.

The concept of ratio can be difficult to grasp. Using counters and putting them into groups to represent the required ratio, then adding up the totals can be helpful for students.

Question 8: Equivalences

In this question six numbers are given: two are percentages, two are fractions and two are decimals. Students are asked to order them with the smallest first.
In order to do this students need to find equivalent decimals (or percentages) for the other values. Students need to understand and use the equivalences between fractions, decimals and percentages.

Students can practice this by sorting a variety of given values and then move on to working out the values themselves. This is a skill which can be practised when doing other parts of the curriculum if the answer is required to be given in a particular form.

In the classroom, problems such as ‘Which is biggest, 60% of £100 or 0.7 of £100 or $\frac{1}{5}$ of £100?’ can stimulate interesting discussions.

Question 9: Advertising

This task presents a pie chart and a table of values showing percentages and angles in the pie chart. Students are asked to complete the table which has one of the percentages and two of the angles missing.

There are many ways of solving this task, but perhaps the easiest ways are:

- Find the percentage spent on Clothes by adding the three given percentages and subtracting the total from 100.
- Find the angle for Cars by calculating 20% or $\frac{1}{5}$ of 360°.
- Then find the angle for Other by making the total of the four angles in the pie chart 360°.

Students need to understand the relationship between angles in the pie chart and percentages. They also need to be able to draw and use accurately drawn pie charts as well as to calculate values using the table when the pie chart is not drawn accurately.

Question 10: Salaries

This question is about averages. Given a table showing the salaries for various groups of people, students are asked to find the modal salary (part a) and say what will happen to the values of the mean and median if one of the salaries increases (part b).

Students have difficulty remembering which average is which so an ‘aide-memoire’ can be useful. Students first learn how to find or calculate the required average but a deeper understanding is required in part b, in order to know what will happen if a value is changed.

In the classroom, questions such as this, followed by discussion and explanation, can develop deeper student understanding.

Question 11: Pentagon

A pentagon with one line of symmetry is shown in this question. The size of two of the angles is given and the question asks students to calculate the size of a third angle. Students are asked to show their work.

This question can be approached in several different ways, but whichever method is chosen the pairs of equal angles, because of the line of symmetry, must first be recognised.

One method would require the student to know or calculate the total for the interior angles of a pentagon. Alternatively, knowing the total for the exterior angles and that the interior angle plus the exterior angle total 180° is an alternative method.

Other useful methods split the pentagon, either down the line of reflection into two identical quadrilaterals, or by drawing a horizontal line to make a triangle and an isosceles trapezium.

Many geometry problems can be solved using several methods. Interesting discussions in the classroom emerge when challenging different groups to find different solutions, maybe giving them a rule which must be used, for example: 'the sum of the angles in a triangle is 180°' or 'the sum of the exterior angles of a polygon is 360°'.

Question 12: Spinning

In this probability question we have two spinners with different numbers, and a chart showing some of the possible totals which can be scored when spinning both spinners. Students are asked to complete the chart to show all the possible totals (part a), and then work out the probability of getting an even number total (part b).

Part a requires an understanding of two-way tables and a little care when doing simple addition. In order to find a correct answer to part b students need to read carefully the requirements for winning, and then calculate the probability using the appropriate algorithm.

Situations such as this can provide challenging classroom problems. Having found the probability of winning, students may find it interesting to decide how much should be charged per 'go', and what size the prize should be in order to encourage lots of people to have a 'go' and still make money for charity on the game.

Question 13: Bronze

This question gives the ratio of copper to tin in bronze. Students are asked to work out the weight of tin in a given weight of bronze (part a), and the total weight of a bronze bracelet that contains a given weight of copper (part b).

Students need to be able to calculate using ratios. Many students find this a difficult topic. Adding a 'total' number to the given ratio, and realizing that the ratio of copper to tin to bronze is 3:1:4 can be helpful, as can thinking that the required numbers can be found using a scale factor.

Question 14: Equations

In this question we are given a pair of simultaneous equations. Students are asked to select the correct answer from a list of five pairs of values.

Two different methods for finding the correct answer to this problem are offered here.

The first method uses substituting the given values for *x* and y in both equations to see whether we get true statements. Use of this method can lead students to use trial and error whenever they have simultaneous equations to solve.

The second method is an algebraic substitution method which can be used when no possible answers are listed. As both equations are in the form *y* = an expression in *x*, the two expressions must be equal so an equation in *x* can be formed and solved to get the correct value for *x*. This can then be substituted into either of the equations (in this case the second is simpler as it doesn't have a fraction) to find *y*. This method works for all pairs of linear equations given in this form and can lead on to solving a linear and quadratic pair of equations.

An interesting classroom activity is to choose a pair of equations which don't have whole number answers and ask half the class to try to solve it using trial and error and the other half to solve it using algebraic substitution.

Question 15: Graphs

In this task, students are given four linear equations and a diagram showing three straight lines drawn on the coordinate x/y plane. The first part of the task asks students to write the correct equations on the three lines (part a). The second part of the task is to draw and label the fourth line on the graph.

One possible solution is described here.

Two of the lines drawn on the graph have a positive gradient. One of these two lines (the blue line) passes through the origin. The equation $y = \frac{1}{4}x$ is the line that matches the blue line.

The second line with a positive gradient intercepts the y-axis at the point $y = 4$. The red line passes through the point (0, 4). The red line matches the equation $y = x + 4$.

The third line drawn on the graph (the green line) has a negative gradient. The only equation with a negative gradient is $y + x = 4$ ($y = -x + 4$), so this is the green line.

The equation $y = x - 4$ is left over and needs to be drawn on the graph.

A line passing through the points (0, -4) and (4, 0) can be drawn. As a check, we can see that this line is parallel to the line $y = x + 4$ (the red line).

Some students do not find it easy to identify the equations of lines that have been drawn. Particularly, when more than one graph has been drawn on the same axes. It is important to encourage students to discuss the properties of graphs after they have drawn them and, when given an equation, try to visualize what the graph will look like before drawing it.

Question 16: Photographs

This task concerns scale factors and the ratio of areas when the size of a photograph is reduced. Four photographs of the same size are to be reduced in size to fit all four onto of one of the originals. The new measurements are to be found (part a), the scale factor of the reduction is asked for (part b) and the ratio of the area of the original to the new area of a photograph is to be worked out (part c).

In order to answer part a of this question, the original measurements need to be halved. In part b, students need to understand that the scale factor relates to the change in length and since the photograph gets smaller this will be a fractional scale factor, $\frac{1}{2}$ rather than 2.

If students find the ratios of original areas and new areas they should be encouraged to simplify these so that they begin to realise that the answer can be found more simply by looking at how many new photographs fit into the area of an original photograph. Alternatively, it may be that students understand that the ratio of the area is the square of the ratio of the length.

Question 17: Cycling Holiday

In this problem, students are asked to work out the distance travelled given the time taken and average speed (part a), and the time needed to travel a given distance at a given speed (part b).

An understanding that, for example, miles per hour means the miles travelled in one hour can help students work out whether to multiply or divide when working with speed, distance and time. A line/road drawn and divided into pieces with hours (1, 2, 3...) above and distance (12, 24, 36...) below can be a useful pictorial representation.

Remembering that Speed = $\frac{\text{Distance}}{\text{Time}}$ can be useful when solving problems such as this.

Question 18: Boxes

This is a question about number sequences. A diagram showing the nets of three open boxes is provided. A table showing the number of squares in two nets, is given and students are asked to complete the table for three and four nets (part a). Students then need to work out the number of squares in the 10th net (part b) and the formula for the number of squares in the *n*th net (part c).

Students may find it difficult to visualise what the 10th net looks like, so working step by step from the given diagrams and the table is often useful. Discussion of how the diagrams change as they get bigger, and what the 10th net looks like without listing all the numbers in between can help students understand the process. In this problem, we can see that the 10th net has a square base measuring 10 by 10, and four sides each measuring 1 by 10. From this we can see that the *n*th net has a square base measuring *n* by *n* and four sides measuring 1 by *n*. This gives us a formula *n2* + 4*n.*

Question 19: Driving Schools

This problem compares the pass rate of two driving schools. Students need to calculate the number of people who passed at the first driving school given the percentage pass rate and the total number of student drivers (part a), and then calculate which school had the best results (part b).

The first part requires the student to work out 70% of 50 so an understanding that percent means per hundred and that 50 is half of 100 is key to understanding the easiest way to working this out.

In the second part, students need to realise that comparison must be made between either the percentage or the fraction of students that passed, not between the number of passes, as the number of students at the two driving schools was different.

Calculating 42 out of 75 gives us 56%, so DriveRight had the best results last year.

Question 20: Cuboid

This question is about using the formulae for the total lengths of the edges and the length of the internal diagonal of a cuboid. The formulae are given along with a diagram to show clearly what the letters in the formulae refer to. The question asks the students to calculate the total length of the edges (part a), and the diagonal length (part b) for a cube of edge length 3cm.

Although the formulas are given, students must first realise that a cube is just a special cuboid with all three dimensions the same, so in this question a, b and c are all equal to 3. Students at this stage will probably not be familiar with these formulas but they should be able to substitute into them to calculate the required answers. It is not only necessary to substitute into the formulas correctly, but the operations need to be done in the correct order.

The final step in part b is to give the answer correct to one decimal place.

Question 21: Mistakes

This task gives students an opportunity to 'be the teacher' correcting an incorrect long multiplication (part a) and a long division (part b).

The error in the long multiplication is one which many students make. When multiplying by 20, they forget to write a 0 in the units column, so actually multiply by 2. This is a conceptual error which students need to understand.

The long division has a simple subtraction error. Many students struggle to be able to do long division and to find this error, they need to know how to do the process correctly.

Correcting incorrect calculations is a useful way of practising number work. Practising long multiplication and division when doing other parts of the curriculum can help to keep this skill fresh, rather than only revising it as a separate topic.

Question 22: Mowing a Lawn

This geometry task requires the use of Pythagoras' Theorem to find one of the sides of a triangle. Students need to read, and understand, the story about mowing a lawn with an electric mower, and realise that they need to consider the triangle AEP even though the line PE is not drawn on the given diagram.

Students are asked to show their working since credit can be awarded for an incorrect answer which shows the correct use of Pythagoras' Theorem (16*2* – 82). Students need practice at solving problems such as this. Working out the length of sides on diagrams, even when all that is needed is a subtraction (to find PA), is something some students find very difficult, especially if they need to interpret information given as a story.

Question 23: Bigger or Smaller

Here the students are given four expressions in *n* and asked, given that *n* is greater than zero, whether the expression is bigger or smaller than *n* or if it could be either depending on the value of *n*.

The answer for the first expression is given.

The second expression multiplies *n* by 0.01. Students often assume that multiplication always gets an answer which is bigger, but this is not the correct answer here.

The third expression multiplies *n* by 100. This will probably be answered correctly by almost everyone.

The fourth expression divides *n* by 0.1. Another misconception, that all divisions produce a smaller answer, is being challenged here.

In order to answer these questions correctly, students will find it helpful to substitute different values for *n* in each of the given expressions.

Calculations using numbers other than positive numbers greater than 1 can surprise many students and should be experimented with.

Finding rules for them can be an interesting project.

Digital Test

Question 1: Quarters

This question asks students to write 1/4 as a percentage (part a), what angle is a 1/4 turn (part b) and work out 1/4 of 6 (part c).

Using fractions in different contexts provides an opportunity for students to recognise the connections between fractions, percentages, angles and mixed numbers.

Classroom activities making up questions using other fractions, such as 1/3 , within different contexts can provide further insights into these connections.

Question 2: Ordering

In this question students are asked to order numbers with up to three decimal places from smallest to largest. The numbers have one, two or three decimal places, so students need to understand that they should start by comparing the first decimal digit, then the second etc.

Students may find it helpful to add extra zeros at the end of some of the decimal numbers so that all the numbers have the same number of digits after the decimal point.

In the classroom, asking a variety of questions such as 'Which number has 6 in the hundredths position?' can help students to develop a deeper understanding of place value.

Question 3: Cubes

This question shows a diagram of a solid made using centimetre cubes: students are asked to find the volume of the solid. In order to answer this question, students need to understand that each cube has a volume of 1cm3 and, therefore, the volume is the same as the number of cubes.

Some students find it very difficult to interpret drawings of three dimensional shapes, so classroom activities involving making solids that correspond to diagrams using actual cubes can be extremely helpful.

Question 4: Brackets

In this question students are asked to add brackets to both sides of an equation, so that they work out to the same value.

Students need to be able to use brackets appropriately.

The use of further classroom activities such as this illustrates the importance of knowing and using the correct order of operations and practising multiplication tables.

Questions 5 and 6: Decimal Cards

Seven cards are provided on which are written brackets, operations and numbers.

Students are asked to order the cards to produce calculations which result in given answers. Although this question is similar to the last question, it is more difficult because some of the numbers are decimals.

Making up calculations using number cards can be an interesting activity in the classroom. Given a fixed set of cards, students can be asked to find the calculation which gives the biggest answer, the smallest answer, a prime number answer etc.

Questions 7, 8 and 9: Sale Prices

This question shows a graph comparing the original price with the sale price.

Students are asked to complete the labels on three items, so that each label shows the original price and the sale price.

The first label requires the sale price to be found given the original price but the second label is the opposite way round, it gives the sale price and the original price needs to be found, so care needs to be taken reading from the correct axis.

The scales themselves need careful reading as a large square on each axis represents £20, so each small square represents £4.

The last label has an original price which is outside the values on the graph so a different value needs to be found and scaled up to get the required answer.

Reading off values from all sorts of graphs with different scales is an important skill and needs to be practised in the classroom.

Question 10: Day Care

In this problem students need to divide a large number in a given ratio. They are asked to choose the two correct calculations and then work out the answer.

Calculating with large numbers, such as the one in this question, can worry some students, as often they practice using ratios with smaller values. Once an understanding of the concept of ratio has been established it is useful to solve problems with all sorts of numbers.

The concept of ratio can be difficult to grasp. Using counters and putting them into groups to represent the required ratio, then adding up the totals can be helpful for students.

Question 11: Equivalences

In this question six numbers are given: two are percentages, two are fractions and two are decimals. Students are asked to order them with the smallest first.

In order to do this, students need to find equivalent decimals (or percentages) for the other values. Students need to understand and use the equivalences between fractions, decimals and percentages.

Students can practice this by sorting a variety of given values and then move on to working out the values themselves. This is a skill which can be practised when doing other parts of the curriculum if the answer is required to be given in a particular form.

In the classroom, problems such as 'Which is biggest, 60% of £100 or 0.7 of £100 or 1/5 of £100?' can stimulate interesting discussions.

Question 12: Advertising

This task presents a pie chart and a table of values showing percentages and angles in the pie chart. Students are asked to complete the table which has one of the percentages and two of the angles missing.

There are many ways of solving this task, but perhaps the easiest ways are:

- Find the percentage spent on Clothes by adding the three given percentages and subtracting the total from 100.
- Find the angle for Cars by calculating 20% or 1/5 of 360°.
- Then find the angle for Other by making the total of the four angles in the pie chart 360°.

Students need to understand the relationship between angles in the pie chart and percentages. They also need to be able to draw and use accurately drawn pie charts as well as to calculate values using the table when the pie chart is not drawn accurately.

Questions 13 and 14: Salaries

This question is about averages. Given a table showing the salaries for various groups of people, students are asked to find the modal salary (question 13) and say what will happen to the values of the mean and median if one of the salaries increases (question 14).

Students have difficulty remembering which average is which so an 'aide-memoire' can be useful. Students first learn how to find or calculate the required average but a deeper understanding is required in question 14, in order to know what will happen if a value is changed.

In the classroom, questions such as this, followed by discussion and explanation, can develop deeper student understanding.

Question 15: Pentagon

A pentagon with one line of symmetry is shown in this question. The size of two of the angles is given and the question asks students to choose the fact that you could use to calculate the size of a third angle and to then give the answer.

This question can be approached in several different ways, but whichever method is chosen the pairs of equal angles, because of the line of symmetry, must first be recognised.

One method would require the student to know or calculate the total for the interior angles of a pentagon. Alternatively, knowing the total for the exterior angles and that the interior angle plus the exterior angle total 180° is an alternative method.

Other useful methods split the pentagon, either down the line of reflection into two identical quadrilaterals, or by drawing a horizontal line to make a triangle and an isosceles trapezium.

Many geometry problems can be solved using several methods. Interesting discussions in the classroom emerge when challenging different groups to find different solutions, maybe giving them a rule which must be used, for example: 'the sum of the angles in a triangle is 180°' or 'the sum of the exterior angles of a polygon is 360°'.

Question 16: Spinning

In this probability question we have two spinners with different numbers, and a chart showing some of the possible totals which can be scored when spinning both spinners. Students are asked to complete the chart to show all the possible totals (part a), and then work out the probability of getting an even number total (part b).

Part a requires an understanding of two-way tables and a little care when doing simple addition. In order to find a correct answer to part b students need to read carefully the requirements for winning, and then calculate the probability using the appropriate algorithm.

Situations such as this can provide challenging classroom problems. Having found the probability of winning, students may find it interesting to decide how much should be charged per 'go', and what size the prize should be in order to encourage lots of people to have a 'go' and still make money for charity on the game.

Question 17: Bronze

This question gives the ratio of copper to tin in bronze. Students are asked to work out the weight of tin in a given weight of bronze (part a), and the total weight of a bronze bracelet that contains a given weight of copper (part b).

Students need to be able to calculate using ratios. Many students find this a difficult topic. Adding a 'total' number to the given ratio, and realizing that the ratio of copper to tin to bronze is 3:1:4 can be helpful, as can thinking that the required numbers can be found using a scale factor.

Question 18: Equations

In this question we are given a pair of simultaneous equations. Students are asked to select the correct answer from a list of five pairs of values.

Two different methods for finding the correct answer to this problem are offered here.

The first method uses substituting the given values for x and y in both equations to see whether we get true statements. Use of this method can lead students to use trial and error whenever they have simultaneous equations to solve.

The second method is an algebraic substitution method which can be used when no possible answers are listed. As both equations are in the form y = an expression in x, the two expressions must be equal so an equation in x can be formed and solved to get the correct value for x. This can then be substituted into either of the equations (in this case the second is simpler as it doesn't have a fraction) to find y. This method works for all pairs of linear equations given in this form and can lead on to solving a linear and quadratic pair of equations.

An interesting classroom activity is to choose a pair of equations which don't have whole number answers and ask half the class to try to solve it using trial and error and the other half to solve it using algebraic substitution.

Question 19: Graphs

In this task, students are given four linear equations and a diagram showing three straight lines drawn on the coordinate x/y plane. The first part of the task asks students to move the correct equations to the three lines (part a). The second part of the task is to draw and label the fourth line on the graph.

One possible solution is described here.

Two of the lines drawn on the graph have a positive gradient. One of these two lines (the blue line) passes through the origin. The equation y = 1/4 x is the line that matches the blue line.

The second line with a positive gradient intercepts the y-axis at the point y = 4. The red line passes through the point (0, 4). The red line matches the equation $y = x + 4$.

The third line drawn on the graph (the green line) has a negative gradient. The only equation with a negative gradient is $y + x = 4$ ($y = -x + 4$), so this is the green line.

The equation $y = x - 4$ is left over and needs to be drawn on the graph.

A line passing through the points (0, -4) and (4, 0) can be drawn. As a check, we can see that this line is parallel to the line $y = x + 4$ (the red line).

Some students do not find it easy to identify the equations of lines that have been drawn. Particularly, when more than one graph has been drawn on the same axes. It is important to encourage students to discuss the properties of graphs after they have drawn them and, when given an equation, try to visualize what the graph will look like before drawing it.

Question 20: Photographs

This task concerns scale factors and the ratio of areas when the size of a photograph is reduced. Four photographs of the same size are to be reduced in size to fit all four onto of one of the originals. The new measurements are to be found (part a), the scale factor of the reduction is asked for (part b) and the ratio of the area of the original to the new area of a photograph is to be worked out (part c).

In order to answer part a of this question, the original measurements need to be halved. In part b, students need to understand that the scale factor relates to the change in length and since the photograph gets smaller this will be a fractional scale factor, 1/2 rather than 2.

If students find the ratios of original areas and new areas they should be encouraged to simplify these so that they begin to realise that the answer can be found more simply by looking at how many new photographs fit into the area of an original photograph. Alternatively, it may be that students understand that the ratio of the area is the square of the ratio of the length.

Question 21: Cycling Holiday

In this problem, students are asked to work out the distance travelled given the time taken and average speed (part a), and the time needed to travel a given distance at a given speed (part b).

An understanding that, for example, miles per hour means the miles travelled in one hour can help students work out whether to multiply or divide when working with speed, distance and time. A line/road drawn and divided into pieces with hours (1, 2, 3...) above and distance (12, 24, 36...) below can be a useful pictorial representation.

Remembering that Speed = Distance/Time can be useful when solving problems such as this.

Questions 23, 24 and 25: Boxes

This is a question about number sequences. A diagram showing the nets of three open boxes is provided. A table showing the number of squares in two nets, is given and students are asked to complete the table for three and four nets (question 22). Students then need to work out the number of squares in the 10th net (question 23) and the formula for the number of squares in the nth net (question 24).

Students may find it difficult to visualise what the 10th net looks like, so working step by step from the given diagrams and the table is often useful. Discussion of how the diagrams change as they get bigger, and what the 10th net looks like without listing all the numbers in between can help students understand the process. In this problem, we can see that the 10th net has a square base measuring 10 by 10, and four sides each measuring 1 by 10. From this we can see that the nth net has a square base measuring n by n and four sides measuring 1 by n. This gives us a formula n2 + 4n.

Question 25: Driving Schools

This problem compares the pass rate of two driving schools. Students need to calculate the number of people who passed at the first driving school given the percentage pass rate and the total number of student drivers (part a), and then calculate which school had the best results (part b).

The first part requires the student to work out 70% of 50 so an understanding that percent means per hundred and that 50 is half of 100 is key to understanding the easiest way to working this out.

In the second part, students need to realise that comparison must be made between either the percentage or the fraction of students that passed, not between the number of passes, as the number of students at the two driving schools was different.

Calculating 42 out of 75 gives us 56%, so DriveRight had the best results last year.

Questions 26 and 27: Cuboid

This question is about using the formulae for the total lengths of the edges and the length of the internal diagonal of a cuboid. The formulae are given along with a diagram to show clearly what the letters in the formulae refer to. The question asks the students to calculate the total length of the edges (question 26), and the diagonal length (question 27) for a cube of edge length 3cm.

Although the formulas are given, students must first realise that a cube is just a special cuboid with all three dimensions the same, so in this question a, b and c are all equal to 3. Students at this stage will probably not be familiar with these formulas but they should be able to substitute into them to calculate the required answers. It is not only necessary to substitute into the formulas correctly, but the operations need to be done in the correct order.

The final step in question 27 is to give the answer correct to one decimal place.

Questions 28 and 29: Mistakes

This task gives students an opportunity to 'be the teacher' correcting an incorrect long multiplication (question 28) and a long division (question 29).

The error in the long multiplication is one which many students make. When multiplying by 20, they forget to write a 0 in the units column, so actually multiply by 2. This is a conceptual error which students need to understand.

The long division has a simple subtraction error. Many students struggle to be able to do long division and to find this error, they need to know how to do the process correctly.

Correcting incorrect calculations is a useful way of practising number work.

Practising long multiplication and division when doing other parts of the curriculum can help to keep this skill fresh, rather than only revising it as a separate topic.

Question 30: Mowing a Lawn

This geometry task requires the use of Pythagoras' Theorem to find one of the sides of a triangle. Students need to read, and understand, the story about mowing a lawn with an electric mower, and realise that they need to consider the triangle AEP even though the line PE is not drawn on the given diagram.

Students are asked to choose the correct fact to calculate the length of AE and to then find the length of AE correct to 2 decimal places.

Students need practice at solving problems such as this. Working out the length of sides on diagrams, even when all that is needed is a subtraction (to find PA), is something some students find very difficult, especially if they need to interpret information given as a story.

Question 31: Bigger or Smaller

Here the students are given four expressions in n and asked, given that n is greater than zero, whether the expression is bigger or smaller than n or if it could be either depending on the value of n.

The answer for the first expression is given.

The second expression multiplies n by 0.01. Students often assume that multiplication always gets an answer which is bigger, but this is not the correct answer here.

The third expression multiplies n by 100. This will probably be answered correctly by almost everyone.

The fourth expression divides n by 0.1. Another misconception, that all divisions produce a smaller answer, is being challenged here.

In order to answer these questions correctly, students will find it helpful to substitute different values for n in each of the given expressions.

Calculations using numbers other than positive numbers greater than 1 can surprise many students and should be experimented with.

Finding rules for them can be an interesting project.

Feedback to parents and carers

A report on the individual student is available to support feedback to parents or carers. This *Individual report for parents* strips away much of the technical detail that is included in the *Group report for teachers*. A series of statements, tailored for parents, is included to explain what the results mean and how learning may be affected. Recommendations focus on how the parent or carer can work with the school to support the student at home.

In addition to the *Individual report for parents*, you may wish to provide supporting information, either orally or in writing, explaining the process and outcomes. The following list provides you with guidelines to assist with this communication.

- Stress the school's commitment to identifying and addressing the needs of each individual student in order to understand and maximise their potential.
- Explain that testing with *PTM*13 is part of the school's regular assessment regime and that all students in the year group(s) have been tested.
- As part of the test, students were tested on their mental maths ability as well as their ability to apply and understand mathematics in a written context.
- You may wish to summarise the specific outcomes and recommendations from the test for that individual student (which are also shown on the *Individual report for parents*).
- Parents or carers should be reassured that if they have any questions or concerns or would like any further advice on how best to support their child, then they should contact the school.

A sample letter (Figure 1) is provided to support your communications with parents/carers after testing with *PTM13*.

Figure 1: Sample parent/carer feedback letter

Dear Parent or Carer,

In school, we wish to assess all our students to see what their needs are and how we can best help them learn and achieve.

As part of this process, your child has completed the Progress Test in Maths 13, which assesses key aspects of maths, such as shape, number and mathematical concepts (like money, place value and time).

A copy of the Individual report for parents is included*. This shows your child's results and describes what these mean in terms of the ways in which he/she will learn best and how you can support him/her at home.

[If the report is not included a relevant short extract can be included instead.]

If you have any queries or concerns please contact us.

Yours faithfully,

[School/Establishment name]

* If possible, it is helpful to parents to discuss the report with them on a suitable occasion before sending it out.