Mathematics process categories   Form A

All of the UK curricula define multiple categories of mathematical proficiency that require students to be able to use and apply mathematics, beyond simple recall of facts and standard procedures. While the intentions are very similar, the terminology varies between regions. *Progress Test in Maths*' *(PTM)* categories are based on the *Curriculum Aims* in the KS1, KS2 and KS3 National Curriculum for England (2013), and are also comparable with the GCSE Assessment Objectives: they adopt some language from both. The main change has been to divide 'Fluency' into two strands.

FF: Fluency in facts and procedures

Students can, for example:

- recall mathematical facts, terminology and definitions (such as the properties of shapes);
- recall number bonds and multiplication tables;
- perform straightforward calculations.

FC: Fluency in conceptual understanding

Students can, for example:

- demonstrate understanding of a mathematical concept in the context of a routine problem (for example, calculate with or compare decimal numbers, identify odd numbers, prime numbers, and multiples);
- extract information from common representations, such as charts, graphs, tables and diagrams;
- identify and apply the appropriate mathematical procedure or operation in a straightforward word problem (for example, knowing when to add, multiply or divide).

MR: Mathematical reasoning

Students can, for example:

- make deductions, inferences and draw conclusions from mathematical information;
- construct chains of reasoning to achieve a given result;
- interpret and communicate information accurately.

PS: Problem solving

Students can, for example:

- translate problems in mathematical or non-mathematical contexts into a process or a series of mathematical processes;
- make and use connections between different parts of mathematics;
- interpret results in the context of the given problem;

- evaluate methods used and results obtained;
- evaluate solutions to identify how they may have been affected by assumptions made.

There is a limit to how thoroughly MR and PS can be assessed in a short, whole-curriculum test such as *PTM*. Teachers are urged to ensure that their curriculum includes a balanced diet of extended tasks, investigations, problem solving and collaborative activities.

These tables show how the questions in *PTM 11T* map onto these process categories.

Paper test		
Process category	**Mental Maths**	**Applying and Understanding Maths**
FF: Fluency in facts and procedures	1, 2, 3, 4, 7, 9, 15, 16, 17, 19	1, 6, 9, 13
FC: Fluency in conceptual understanding	5, 6, 8, 10, 11, 12, 13, 14, 18, 20	2, 4, 5, 10, 11
MR: Mathematical reasoning		3, 7, 8, 15, 16, 18, 19
PS: Problem solving		12, 14, 17, 20

Digital test		
Process category	**Mental Maths**	**Applying and Understanding Maths**
FF: Fluency in facts and procedures	1, 2, 3, 4, 7, 9, 15, 16, 17, 19	1, 10, 11, 15a, 15b, 15c, 23
FC: Fluency in conceptual understanding	5, 6, 8, 10, 11, 12, 13, 14, 18, 20	2a, 2b, 4, 5, 6, 7, 8, 9, 16, 17, 18, 19
MR: Mathematical reasoning		3, 12, 13a, 13b, 14, 25a, 25b, 25c, 26, 28a, 28b, 29a, 29b, 30a, 30b
PS: Problem solving		20, 21, 22, 24a, 24b, 27, 31

Mathematics process categories in Wales, Scotland and Northern Ireland

The process categories shown above are based on the National Curriculum and GCSE syllabuses for England. The curricula for Wales, Scotland and Northern Ireland have similar requirements, although there is wide variation in the way they are defined.

Wales	Closest *PTM* process categories			
Key Stage 2 Phase Skills	**FF**	**FC**	**MR**	**PS**
1. Solve mathematical problems				•
2. Communicate mathematically		•	•	
3. Reason mathematically		•	•	
Key Stage 2 Phase Range	•			

Northern Ireland	Closest *PTM* process categories			
Key Stage 2 Processes in Mathematics	**FF**	**FC**	**MR**	**PS**
Making and monitoring decisions				•
Communicating mathematically		•	•	
Mathematical reasoning		•	•	•
Individual mathematical topics	•			

Scotland	Closest *PTM* process categories			
Experiences and outcomes	**FF**	**FC**	**MR**	**PS**
develop a secure understanding of the concepts, principles and processes of mathematics and apply these in different contexts, including the world of work			•	•
engage with more abstract mathematical concepts and develop important new kinds of thinking			•	
understand the application of mathematics, its impact on our society past and present, and its potential for the future *				
develop essential numeracy skills which will allow me to participate fully in society	•			
establish firm foundations for further specialist learning	•	•		
understand that successful independent living requires financial awareness, effective money management, using schedules and other related skills			•	•
interpret numerical information appropriately and use it to draw conclusions, assess risk, and make reasoned evaluations and informed decisions				•
apply skills and understanding creatively and logically to solve problems, within a variety of contexts			•	•
appreciate how the imaginative and effective use of technologies can enhance the development of skills and concepts				

* Education Scotland "Curriculum for Excellence: Numeracy and Mathematics" 14 May 2009. Accessed: 31 July 2014. www.curriculumforexcellencescotland.gov.uk

Assessment for learning: following up the test activities

Each *PTM* assessment test is designed to align with the mathematics curriculum at a level appropriate for the students in the relevant age group. The activities may therefore be used to obtain diagnostic information about each student's strengths and weaknesses, and may also be used to provide a basis from which students' mathematical understanding may be further developed.

This section discusses some of the ways in which students may be helped to improve areas of weakness and to build on what they already know in order to deepen their understanding. These notes cover only a few of the possibilities. In talking to students and discussing the activities on which they did well, in addition to those they were unable to complete correctly, you may find approaches that are helpful to them, building on their own strengths and interests.

You will need to refer to the activities in the Student Booklet and At a Glance Guide when reading these notes, as they form the basis of the ideas suggested. The activities are referred to here by both their numbers and their names.

Formative notes on the questions

The standardised total scores on *PTM* give you an indication of the overall performance of your students, and a basis for progress monitoring. This section is intended to you help identify the specific difficulties that students have with individual questions, and to suggest possible activities to help guide your future teaching.

Mental Maths test

These questions test students' basic number skills and recall of facts. If students score poorly, it may be that they simply lack these skills and are relying too heavily on written methods for even simple methods. They may lack the confidence to recall mathematical facts under pressure.

Regular quick-fire quizzes may help students gain fluency and confidence, and there are many software packages that allow students to practice skills in the context of games.

However, these should not displace problem-solving and investigative mathematics activities, which can also help students gain fluency by fostering a deeper understanding of mathematical concepts and their connections, reducing their dependence on 'memorising' fragments of information.

Paper Test

Question 1: Number sequence

This task asks students to complete a number sequence in which the numbers go down in fives. The third number to be filled in is a negative number. This requires students to count backwards through zero to include negative numbers.

The use of a number line can be valuable when dealing with sequences. Looking for patterns in sequences should be encouraged, for example in the multiplication tables. When students find negative numbers difficult, simply counting backwards starting from, say, 10 can be a useful activity. It can also be very helpful to point out situations in real life where negative numbers occur: for example, on a thermometer (as in Question 5) and when using heights above and below sea level.

Question 2: Distances from school

In this question students are presented with a problem which involves comparing fractions with different denominators. Students need to recognise that the denominators are multiples of two and compare equivalent fractions.

The use of practical equipment can aid understanding. Real-life situations such as sharing a pizza can be helpful. Questions such as, 'Do you get more if you have $\frac{3}{4}$ or $\frac{5}{8}$ of the pizza?' and 'If you cut your pizza into eight equal pieces, how many pieces will you get if you have $\frac{1}{4}$ of the pizza? '

A card game of 'Snap' can be made by the students themselves. This makes an enjoyable group/paired activity. Each group, or pair, creates a set of 'Snap' cards with fractions such as $\frac{1}{4}$, $\frac{4}{8}$, $\frac{1}{2}$, $\frac{2}{8}$, $\frac{3}{4}$, $\frac{6}{8}$, $\frac{4}{4}$ and so on, and then they can play the game. The discussion involved when creating a set of cards for a game is very worthwhile and fun.

Question 3: Nails

This task requires students to solve a two-step multiplication problem which results in a five-digit number.

Enjoyable activities to practice and extend such skills, without the use of a calculator, may ask students to calculate, for example, how many seconds there are in a week; how many days they themselves, or their parents, have been alive; how many people can be seated in a multiscreen cinema where there are four small cinemas each with thirty rows of 60 seats, etc.

Question 4: Pattern

Here students are presented with a square grid on which six L-shapes are drawn. The task requires students to understand reflective (part a) and translational symmetry (part b).

A follow-up art activity which is helpful in reinforcing such learning is to use squared paper to create symmetrical patterns and then colour them in. It's a good idea to ask the children to fold their paper into half horizontally and then vertically into quarters, so that they have two axes with which to begin creating their patterns. Cutting out shapes on folded paper is also very illuminating. Symmetrical patterns can be found in folk art, Islamic designs and Rangoli patterns and they can make a very attractive, thought-provoking display.

Question 5: Hotter or colder

This task asks students to read and interpret a table which records monthly temperatures. They are asked four questions, one of which requires them to understand and calculate with negative numbers.

Learning to read a real thermometer is a useful life skill. Students can gather their own data about the weather, create tables and then pose questions for their peers to answer. Ordering a set of temperatures, for example putting the lowest first, will increase students' confidence. Questions such as, 'Which temperature is lower –6°C or –1°C?' and 'If the temperature rises by 12°C from –5°C what will the temperature be?' will help students to understand negative numbers. Data can be gathered from the internet on world weather and other climates, so that students can pose problems to be answered by their peers.

Question 6: Percentages

This problem requires students to recognise the shaded percentages in a series of squares (part a) and to understand that 25% is equivalent to $\frac{1}{4}$ or 0.25 (part b).

When students are not yet secure about the concept of percentage it is useful to produce a 100 square and have them colour in various percentages, for example 20% in blue, 50% in red, 2% in yellow 28% in purple. This activity can be varied in many ways. It can represent the different percentages of use in a park: 10% children's playground, 40% tennis courts, 30% fields for games, 20% gardens.

It's worth unpacking the word 'percentage' by explaining that it comes from the Latin 'per centum' which translates as 'by the hundred'. Students can then find other words that have the root 'cent' in them, and which refer to a hundred: centigrade, centimetre, century, centenarian, centennial, and so on.

The game of 'Snap' which was described in the activities suggested for Question 2 could now have percentage cards added to the pack to enrich the game and strengthen students' understanding of fraction and percentage equivalents.

Question 7: Sun

The question states, in words, the distance of the Earth from the Sun and asks students to write this distance in figures.

Writing 150,000,000 may be difficult for some students, so practice in writing large numbers such as populations of different countries or the chances of winning the lottery is useful.

Students need to say and write numbers correctly and with precision, so that they are clear about place value. Some students will benefit from practicing reading and saying the place value of each digit when using very large numbers.

Question 8: Currency

Here students are required to read and interpret a graph which converts British pounds to euros. First, students are asked to solve two problems based on the graph (parts a and b). After being told that £100 is equal to 150 Australian dollars, the final task (part c) is to draw a second line on the graph which can be used to convert British pounds to Australian dollars.

Students should understand and use a range of graphs such as temperature, distance–time and currency conversion and apply their knowledge in science and other subjects. Opportunities can be found for drawing and using such graphs using data gathered in sporting activities such as running races and from everyday situations that can be found on the internet.

Question 9: Signs

This question requires students to insert the operation signs =, + or – in the empty boxes in order to make the three calculations correct.

Students enjoy puzzles such as these missing number/operation problems. A worthwhile activity could be to allow them to work in groups to devise similar puzzles using different numbers or operations of their own. Pre-algebra problems like this can be extended to simple equations using symbols instead of numbers. When they are more confident, students can be shown how to build an algebraic expression. Later they can attempt simple problems such as: If $10 + y = 15$ find the value of y.

Puzzles such as magic squares and Sudoku can also help with number fluency.

Question 10: Sixty

Here students are presented with five calculations involving fractions of 60, four of which have the same answer. They need to circle the odd one out. In order to select the odd one out students need to understand the equivalence of percentages and fractions.

It is worth discussing this task as a whole-class activity and showing all of the working for those students who find it difficult. Similar problems can then be set for numbers such as 100, 50, 40, and so on.

Question 11: Newspapers

A graph which shows some data about newspaper sales is presented and students are asked to answer three questions: ‘Which is the most popular?’ (part a), ‘Which is the least popular?’ (part b), and ‘Which three newspapers sell more than a million copies a day?’ (part c).

More practice in constructing, reading and interpreting graphs which use very large numbers will be helpful. Here the challenge will be to get the scale correct. Lots of suitable data, such as populations of countries or food consumption, can be collected from the internet.

Question 12: Same area

This question deals with area and perimeter. Four shapes are drawn on a square grid; three have the same area. First, students are asked to identify the three shapes with the same area (part a). They are then asked to identify the three shapes which have the same perimeter (part b). Finally, they are asked to draw, on the grid, a different shape that has the same area as shape B (part c). Tasks such as the final part of this question are not frequently posed.

Although area and perimeter seem to be very straightforward concepts, many students get them confused. It is useful to teach the two concepts separately. Lots of practical classroom work is needed. Students should be introduced to the concept of area, initially by counting squares and, only later, use perimeter measurements to calculate areas.

Question 13: Number of students

The task begins with the statement that the Park School has 1500 students, to the nearest 100. There follows six possible numbers which could be the correct number of students in this school. The task is to identify and circle these numbers.

Students need to be able to round up and round down to the nearest 100.
A pleasant activity which can help here is for the teacher to write on the board about twenty random numbers between 5 and 99. Along the bottom of the board each of the tens (10-100) is drawn in a coloured circle. Students then take it in turns to come to the board and select and colour in one of the numbers in the correct colour, to indicate what it is rounded to the nearest ten. This can then be extended to rounding with hundreds. Students should be reminded that 5 is rounded up not down.

Another useful activity is to divide the board into four columns with the following headings and then ask the students to come, in turn, to the board and round each number to the nearest 100. For example:

Chosen number	Round up	Round down	Rounded number
6981	✓		7000
2024		✓	2000

Question 14: Tickets

This task deals with the proportion of profit Ali makes on the sale of concert tickets. Initially, it may be that students calculate that he makes a profit of £1 for every £20 of sales. The first part of the question (part a) is to calculate how much he makes with the sale of a £60 ticket, and then how much the ticket sales would be if Ali received £6 (part b). Alternatively, students may work with the notion that the ratio of profit to cost is 2:40 or 1:20.

The concept of proportion can sometimes be challenging and lots of practical work is often needed. It is important to ensure that students understand that ratio compares one amount with another and to begin simply with everyday examples such as looking at the proportions of ingredients in a recipe for biscuits or jam. For example, if 1kg of sugar and 800g of raspberries are needed to make two jars of jam, what would be the proportions and quantities needed to make six jars of jam.

Question 15: School hall

This question states that there are 60 students in the school hall and that 1 in 4 of them have a packed lunch; $\frac{1}{3}$ of them live within 500 meters of the school; and 10% of them wear glasses. Students are then asked to calculate the number of students who fall into each of these categories. Here students need to think flexibly using their understanding of proportion, fractions and percentages within the same every day situation.

It could be worth 'unpacking' this sort of problem as a whole-class activity since students may find it challenging. More practice in using several concepts within a single task will be helpful here. A similar question could be devised using real classroom situations, such as the colour of their sweatshirts/shoes/ bags, whether they have pets etc.

Question 16: Angles at a point

The diagram here shows two angles which together make a complete turn. The question requires the students to know that there are 360° in a complete turn and begin by dividing 360 by 3 and then share out the degrees in the proportions 2:1.

Lots of practice in using a protractor correctly will ensure students are confident in measuring angles in a variety of shapes. Activities such as cutting out a triangle, tearing off the three corners and then pasting them on a straight line can help students understand that the three angles in a triangle add up to 180°. After students understand this, they can be given two of the angles in a triangle and asked to calculate the third angle.

Question 17: Jug

In order for the students to be successful in solving this problem correctly, they need to know that there are 1000 cubic centimetres in a litre of water. The question states that Maya has a two-litre jug of water from which she fills six glasses, each of which hold 250 cubic centimetres. Students then need to say how much water is left in the jug, and show their calculation.

First, it is worth making sure the students understand that 1 litre of water is 1000 millilitres. Some students may have been confused by the use of the term 'cubic centimetres'. It is necessary for them to understand that volume and capacity are linked and so both are often used to describe capacity. It may be useful to work through the problem with the class reminding them that a 10 cm cube will hold 1 litre (1000 cm^3). The class can then measure other containers such as a box which holds one litre of fruit juice, bearing in mind that the volume is width multiplied by height multiplied by depth and that 1000 cm^3 = 1 litre.

Question 18: Photo frames

Students are told that when Sam frames photographs he charges £20 an hour for labour and he multiplies the cost of the materials used by four. The task is to find the cost when Sam takes half an hour to frame a photograph and the materials used cost £6. Students also need to show their calculations.

As a follow-up activity students could be given a range of similar situations; for example, a plumber who charges a call out fee of £40 and then £30 an hour for labour, and doubles the price of materials used.

Question 19: Flag

Here students are presented with a diagram showing a flag drawn on a four-quadrant grid. First, they are asked to draw the flag after it has moved three squares to the right (part a). Next they are asked to give the new co-ordinates (part b). Then they are told that the horizontal axis is a mirror line and asked to draw the original flag after it has been reflected (part c). The final task asks students to write down the co-ordinates of the mirror image (part d).

Students need to realise that in a grid with four quadrants the horizontal axis has been extended to the left of the central point 0 and the numbers become negative co-ordinates. Similarly, when the vertical axis is extended down we have negative co-ordinates.

Students need to be reminded when working out a position that they must always begin with the origin: this is where the two axes intersect at 0. There are lots of fun ways of remembering the order of writing the co-ordinates; for example, go along the runway and then go up in the air; walk along the house and then put the ladder up to the window; run along the corridor and then up the stairs. Students can invent their own ways of remembering this.

Students find it interesting to draw a variety of shapes with given co-ordinates and also attempt to visualise the shapes that will be produced before drawing them.

Question 20: Bigger than 10

This question begins with the statement: '*N* is a number bigger than 10'. There follows four statements:

- *N* divided by 2 is less than 10
- *N* times 10 is more than 100
- *N* minus 10 is negative
- half of *N* is less than 10.

Students are asked to decide whether they think that each statement must be true, could be true, or must be false.

If students find this task difficult, it is useful to approach the task from the point of view that *N* is a mystery number, but that it needs to be 11 or more. They can then try out the calculations with 11 and then with other larger numbers of their choice.

To help students to become skilled at solving logic problems and puzzles they need to acquire a range of skills, such as:

- working from what is known to what is not known
- using thinking that includes inferences and deductions and testing these
- taking out a piece of information and changing it whilst keeping everything else the same to see how this affects the problem.

This type of thinking can start with very simple tasks.

Digital Test

Question 1: Number sequence

This task asks students to complete a number sequence in which the numbers go down in fives. The third number to be filled in is a negative number. This requires students to count backwards through zero to include negative numbers.

The use of a number line can be valuable when dealing with sequences. Looking for patterns in sequences should be encouraged, for example in the multiplication tables. When students find negative numbers difficult, simply counting backwards starting from, say, 10 can be a useful activity. It can also be very helpful to point out situations in real life where negative numbers occur: for example, on a thermometer (as in questions 6, 7, 8 and 9) and when using heights above and below sea level.

Question 2: Distances from school

In this question students are presented with a problem which involves comparing fractions with different denominators. Students need to recognise that the denominators are multiples of two and compare equivalent fractions.

The use of practical equipment can aid understanding. Real-life situations such as sharing a pizza can be helpful. Questions such as, 'Do you get more if you have ¾ or 5/8 of the pizza?' and 'If you cut your pizza into eight equal pieces, how many pieces will you get if you have 1/4 of the pizza? '

A card game of 'Snap' can be made by the students themselves. This makes an enjoyable group/paired activity. Each group, or pair, creates a set of 'Snap' cards with fractions such as 1/4, 4/8, 1/2, 2/8, 3/4, 6/8, 4/4 and so on, and then they can play the game.

The discussion involved when creating a set of cards for a game is very worthwhile and fun.

Question 3: Nails

This task requires students to solve a two-step multiplication problem which results in a five-digit number.

Enjoyable activities to practice and extend such skills, without the use of a calculator, may ask students to calculate, for example, how many seconds there are in a week; how many days they themselves, or their parents, have been alive; how many people can be seated in a multiscreen cinema where there are four small cinemas each with thirty rows of 60 seats, etc.

Questions 4 and 5: Pattern

Here students are presented with a square grid on which six L-shapes are drawn.

The task requires students to understand reflective (question 4) and translational symmetry (question 5).

A follow-up art activity which is helpful in reinforcing such learning is to use squared paper to create symmetrical patterns and then colour them in. It's a good idea to ask the children to fold their paper into half horizontally and then vertically into quarters, so that they have two axes with which to begin creating their patterns. Cutting out shapes on folded paper is also very illuminating. Symmetrical patterns can be found in folk art, Islamic designs and Rangoli patterns and they can make a very attractive, thought-provoking display.

Questions 6, 7, 8 and 9: Hotter or colder

This task asks students to read and interpret a table which records monthly temperatures. They are asked four questions, one of which requires them to understand and calculate with negative numbers.

Learning to read a real thermometer is a useful life skill. Students can gather their own data about the weather, create tables and then pose questions for their peers to answer. Ordering a set of temperatures, for example putting the lowest first, will increase students' confidence. Questions such as, 'Which temperature is lower –6°C or –1°C?' and 'If the temperature rises by 12°C from –5°C what will the temperature be?' will help students to understand negative numbers. Data can be gathered from the internet on world weather and other climates, so that students can pose problems to be answered by their peers.

Questions 10 and 11: Percentages

This problem requires students to recognise the shaded percentages in a series of squares (question 10) and to understand that 25% is equivalent to 1/4 or 0.25 (question 11).

When students are not yet secure about the concept of percentage it is useful to produce a 100 square and have them colour in various percentages, for example 20% in blue, 50% in red, 2% in yellow 28% in purple. This activity can be varied in many ways. It can represent the different percentages of use in a park: 10% children's playground, 40% tennis courts, 30% fields for games, 20% gardens.

It's worth unpacking the word 'percentage' by explaining that it comes from the Latin 'per centum' which translates as 'by the hundred'. Students can then find other words that have the root 'cent' in them, and which refer to a hundred: centigrade, centimetre, century, centenarian, centennial, and so on.

The game of 'Snap' which was described in the activities suggested for Question 2 could now have percentage cards added to the pack to enrich the game and strengthen students' understanding of fraction and percentage equivalents.

Question 12: Sun

The question states, in words, the distance of the Earth from the Sun and asks students to write this distance in figures.

Writing 150,000,000 may be difficult for some students, so practice in writing large numbers such as populations of different countries or the chances of winning the lottery is useful.

Students need to say and write numbers correctly and with precision, so that they are clear about place value. Some students will benefit from practicing reading and saying the place value of each digit when using very large numbers.

Questions 13 and 14: Currency

Here students are required to read and interpret a graph which converts British pounds to euros. First, students are asked to solve two problems based on the graph (question 13 parts a and b). After being told that £100 is equal to 150 Australian dollars, the final task (question 14) is to create a second line on the graph which can be used to convert British pounds to Australian dollars.

Students should understand and use a range of graphs such as temperature, distance–time and currency conversion and apply their knowledge in science and other subjects. Opportunities can be found for drawing and using such graphs using data gathered in sporting activities such as running races and from everyday situations that can be found on the internet.

Question 15: Signs

This question requires students to insert the operation signs =, + or – in the empty boxes in order to make the three calculations correct.

Students enjoy puzzles such as these missing number/operation problems. A worthwhile activity could be to allow them to work in groups to devise similar puzzles using different numbers or operations of their own. Pre-algebra problems like this can be extended to simple equations using symbols instead of numbers. When they are more confident, students can be shown how to build an algebraic expression. Later they can attempt simple problems such as: If $10 + y = 15$ find the value of y.

Puzzles such as magic squares and Sudoku can also help with number fluency.

Question 16: Sixty

Here students are presented with five calculations involving fractions of 60, four of which have the same answer. They need to choose the odd one out. In order to select the odd one out students need to understand the equivalence of percentages and fractions.

It is worth discussing this task as a whole-class activity and showing all of the working for those students who find it difficult. Similar problems can then be set for numbers such as 100, 50, 40, and so on.

Questions 17, 18 and 19: Newspapers

A graph which shows some data about newspaper sales is presented and students are asked to answer three questions: 'Which is the most popular?' (question 17), 'Which is the least popular?' (question 18), and 'Which three newspapers sell more than a million copies a day?' (question 19).

More practice in constructing, reading and interpreting graphs which use very large numbers will be helpful. Here the challenge will be to get the scale correct. Lots of suitable data, such as populations of countries or food consumption, can be collected from the internet.

Questions 20, 21 and 22: Same area

This question deals with area and perimeter. Four shapes are drawn on a square grid; three have the same area. First, students are asked to identify the three shapes with the same area (question 20). They are then asked to identify the three shapes which have the same perimeter (question 21). Finally, they are asked to create, on the grid, a different shape that has the same area as shape B (question 22). Tasks such as the final part of this question are not frequently posed.

Although area and perimeter seem to be very straightforward concepts, many students get them confused. It is useful to teach the two concepts separately.

Lots of practical classroom work is needed. Students should be introduced to the concept of area, initially by counting squares and, only later, use perimeter measurements to calculate areas.

Question 23: Number of students

The task begins with the statement that the Park School has 1500 students, to the nearest 100. There follows six possible numbers which could be the correct number of students in this school. The task is to identify these numbers.

Students need to be able to round up and round down to the nearest 100. A pleasant activity which can help here is for the teacher to write on the board about twenty random numbers between 5 and 99. Along the bottom of the board each of the tens (10-100) is drawn in a coloured circle. Students then take it in turns to come to the board and select and colour in one of the numbers in the correct colour, to indicate what it is rounded to the nearest ten. This can then be extended to rounding with hundreds. Students should be reminded that 5 is rounded up not down.

Another useful activity is to divide the board into four columns with the following headings and then ask the students to come, in turn, to the board and round each number to the nearest 100. For example:

Chosen number	Round up	Round down	Rounded number
6981	✓		7000
2024		✓	2000

Question 24: Tickets

This task deals with the proportion of profit Ali makes on the sale of concert tickets. Initially, it may be that students calculate that he makes a profit of £1 for every £20 of sales. The first part of the question (part a) is to calculate how much he makes with the sale of a £60 ticket, and then how much the ticket sales would be if Ali received £6 (part b). Alternatively, students may work with the notion that the ratio of profit to cost is 2:40 or 1:20.

The concept of proportion can sometimes be challenging and lots of practical work is often needed. It is important to ensure that students understand that ratio compares one amount with another and to begin simply with everyday examples such as looking at the proportions of ingredients in a recipe for biscuits or jam. For example, if 1kg of sugar and 800g of raspberries are needed to make two jars of jam, what would be the proportions and quantities needed to make six jars of jam.

Question 25: School hall

This question states that there are 60 students in the school hall and that 1 in 4 of them have a packed lunch; 1/3 of them live within 500 meters of the school; and 10% of them wear glasses. Students are then asked to calculate the number of students who fall into each of these categories. Here students need to think flexibly using their understanding of proportion, fractions and percentages within the same every day situation.

It could be worth 'unpacking' this sort of problem as a whole-class activity since students may find it challenging. More practice in using several concepts within a single task will be helpful here. A similar question could be devised using real classroom situations, such as the colour of their sweatshirts/shoes/ bags, whether they have pets etc.

Question 26: Angles at a point

The diagram here shows two angles which together make a complete turn. The question requires the students to know that there are 360° in a complete turn and begin by dividing 360 by 3 and then share out the degrees in the proportions 2:1. Lots of practice in using a protractor correctly will ensure students are confident in measuring angles in a variety of shapes. Activities such as cutting out a triangle, tearing off the three corners and then pasting them on a straight line can help students understand that the three angles in a triangle add up to 180°. After students understand this, they can be given two of the angles in a triangle and asked to calculate the third angle.

Question 27: Jug

In order for the students to be successful in solving this problem correctly, they need to know that there are 1000 cubic centimetres in a litre of water. The question states that Maya has a two-litre jug of water from which she fills six glasses, each of which hold 250 cubic centimetres. Students then need to say how much water is left in the jug, and show their calculation.

First, it is worth making sure the students understand that 1 litre of water is 1000 millilitres. Some students may have been confused by the use of the term 'cubic centimetres'. It is necessary for them to understand that volume and capacity are linked and so both are often used to describe capacity. It may be useful to work through the problem with the class reminding them that a 10 cm cube will hold 1 litre (1000 cm3). The class can then measure other containers such as a box which holds one litre of fruit juice, bearing in mind that the volume is width multiplied by height multiplied by depth and that 1000 cm3 = 1 litre.

Question 28: Photo frames

In part a, students are told that when Sam frames photographs he charges £20 an hour for labour and he multiplies the cost of the materials used by four. The task is to find the cost when Sam takes half an hour to frame a photograph and the materials used cost £6. In part b, the task is to find the amount of money this will make for the business.

As a follow-up activity students could be given a range of similar situations; for example, a plumber who charges a call out fee of £40 and then £30 an hour for labour, and doubles the price of materials used.

Questions 29 and 30: Flag

Here students are presented with a diagram showing a flag drawn on a four quadrant grid. First, they are asked to move the flag three squares to the right (question 29a). Next they are asked to give the new co-ordinates (question 29b). Then they are told that the horizontal axis is a mirror line and asked to move the dots to show the flag after it has been reflected in the mirror line (question 30a). The final task asks students to give the co-ordinates of the mirror image (question 30b).

Students need to realise that in a grid with four quadrants the horizontal axis has been extended to the left of the central point 0 and the numbers become negative co-ordinates. Similarly, when the vertical axis is extended down we have negative co-ordinates.

Students need to be reminded when working out a position that they must always begin with the origin: this is where the two axes intersect at 0. There are lots of fun ways of remembering the order of writing the co-ordinates; for example, go along the runway and then go up in the air; walk along the house and then put the ladder up to the window; run along the corridor and then up the stairs. Students can invent their own ways of remembering this.

Students find it interesting to draw a variety of shapes with given co-ordinates and also attempt to visualise the shapes that will be produced before drawing them.

Question 31: Bigger than 10

This question begins with the statement: ‘N is a number bigger than 10’. There follows four statements:

- N divided by 2 is less than 10
- N times 10 is more than 100
- N minus 10 is negative
- half of N is less than 10.

Students are asked to decide whether they think that each statement must be true, could be true, or must be false.

If students find this task difficult, it is useful to approach the task from the point of view that N is a mystery number, but that it needs to be 11 or more. They can then try out the calculations with 11 and then with other larger numbers of their choice.

To help students to become skilled at solving logic problems and puzzles they need to acquire a range of skills, such as:

- working from what is known to what is not known
- using thinking that includes inferences and deductions and testing these
- taking out a piece of information and changing it whilst keeping everything else the same to see how this affects the problem.

This type of thinking can start with very simple tasks.

Feedback to parents and carers

A report on the individual student is available to support feedback to parents or carers. This *Individual report for parents* strips away much of the technical detail that is included in the *Group report for teachers*. A series of statements, tailored for parents, is included to explain what the results mean and how learning may be affected. Recommendations focus on how the parent or carer can work with the school to support the student at home.

In addition to the *Individual report for parents*, you may wish to provide supporting information, either orally or in writing, explaining the process and outcomes. The following list provides you with guidelines to assist with this communication.

- Stress the school's commitment to identifying and addressing the needs of each individual student in order to understand and maximise their potential.
- Explain that testing with *PTM 11T* is part of the school's regular assessment regime and that all students in the year group(s) have been tested.
- You may wish to summarise the specific outcomes and recommendations from the test for that individual student (which are also shown on the *Individual report for parents*).
- Parents or carers should be reassured that if they have any questions or concerns or would like any further advice on how best to support their child, then they should contact the school.

A sample letter (Figure 1) is provided to support your communications with parents/carers after testing with *PTM11T*.

Figure 1: Sample parent/carer feedback letter

Dear Parent or Carer,

In school, we wish to assess all our students to see what their needs are and how we can best help them learn and achieve.

As part of this process, your child has completed the Progress Test in Maths 11T, which assesses key aspects of maths, such as shape, number and mathematical concepts (like money, place value and time).

A copy of the Individual report for parents is included*. This shows your child's results and describes what these mean in terms of the ways in which he/she will learn best and how you can support him/her at home.

[If the report is not included a relevant short extract can be included instead.]

If you have any queries or concerns please contact us.

Yours faithfully,

[School/Establishment name]

* If possible, it is helpful to parents to discuss the report with them on a suitable occasion before sending it out.